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Outsourcing analysis for Reverse Logistics systems: a qualitative study and a Markov decision model

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**Outsourcing analysis for Reverse Logistics systems: A qualitative study
and a Markov decision model**

by

Marco Antonio Serrato García

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Industrial Engineering

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Ames, Iowa
2006

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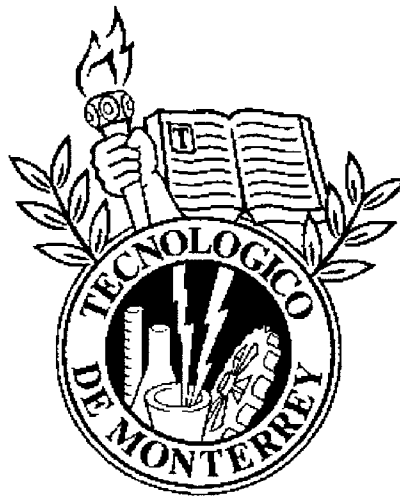
Major Professor

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For the Major Program

INSTITUTO TECNOLÓGICO Y DE ESTUDIOS SUPERIORES DE MONTERREY

CAMPUS TOLUCA



OUTSOURCING ANALYSIS FOR REVERSE LOGISTICS SYSTEMS: A QUALITATIVE STUDY & A MARKOV DECISION MODEL

PRESENTED AS PARTIAL FULFILLMENT FOR THE DEGREE OF:
DOCTOR OF PHILOSOPHY (PhD) IN INDUSTRIAL ENGINEERING

PRESENTED BY:

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DR. JUAN GAYTÁN INIESTRA

Toluca, Mexico.

April 2006

OUTSOURCING ANALYSIS FOR
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TABLE OF CONTENTS

1.	Introduction	1
1.1.	Description of the Methodology	2
1.2.	Introduction to RL Networks	4
1.2.1.	General Characteristics of RL Networks	7
1.2.2.	Factors considered for the RL Networks Categorization	9
1.2.2.1.	Product Life Cycle	10
1.2.2.2.	Variability in the Return Volume	14
1.3.	Research Hypotheses & Objectives	15
2.	Proposed Categorization of RL Networks	17
2.1.	Scenarios Defined	17
2.1.1.	Jet Engines, Airframes and Railroad Locomotive Engines	17
2.1.2.	Ferrous Scrap in the Steel Industry	17
2.1.3.	Hazardous Wastes	18
2.1.4.	Pharmaceuticals	19
2.1.5.	Container Reuse	19
2.1.6.	Tire Remanufacturing	20
2.1.7.	Retailers	21
2.1.8.	Cellular Telephone Reuse	22
2.1.9.	Electronics and Computers	23
2.2.	RL Networks Categorization	24
2.3.	Existing 3PRLP for each Scenario	27
3.	Markov Decision Model	31
3.1.	Model Definition	34
3.1.1.	Decision Epochs	34
3.1.2.	States	34
3.1.3.	Actions	35
3.1.4.	Transition Probabilities	36
3.1.5.	Rewards	36
3.2.	System Dynamics	41
3.3.	Characteristics of the Optimal Policy to be Found	42
3.3.1.	Hypothesis 3 Rewritten	43
3.3.2.	Conditions for Identifying a Monotone Nondecreasing Policy as Optimal	44
3.4.	Requirements in the MDM for the existence of a Monotone Nondecreasing Policy (MNP)	46
3.4.1.	Condition 1	52
3.4.2.	Condition 2	54
3.4.3.	Condition 3	55
3.4.4.	Condition 4	60
3.4.5.	Condition 5	60
3.4.6.	Conclusions on the requirements for a MNP	61
3.5.	Implications for the Suitability of Outsourcing for Scenarios with Higher Return Volume Variability	62
3.5.1.	Numerical Examples	68

4.	Conclusions and Future Work	73
4.1.	Verifying Research Hypotheses	73
4.2.	Future Work	75
	Appendix 1: Matlab Program	77
	Appendix 2: Results from the outputs of the Matlab Program	79
	References	81

1. Introduction.

Once organizations distribute their products to retailers and final consumers, their flow of materials is not over: that is the moment at which the Reverse Logistics (RL) flow begins. These companies are eventually forced to face the enormous amount of problems at the back of their business, which increase significantly when they are not managed correctly.

Because of this reason, RL has recently been considered as an improvement area if it is focused correctly. Every manufacturing, distribution or sales firm, irrespective of its size, types of products or geographic location, can benefit from planning, implementing and controlling RL activities. Unfortunately, not enough analytical models currently exist which assist in RL strategic decisions.

Given the nature of the RL field, one of the most important decisions to be taken by any firm is either to outsource such functions or not. This comes from the fact that RL does not represent a core activity for a firm, given that the purpose of any company is not to manage the flow of products taken back from the sale point, but rather to distribute such products to its customers.

This implies that the outsourcing option for RL is mostly identified as a “take it or leave it” alternative, given that the firm will not be continually changing its strategy to manage such returns. Any organization might decide whether to perform the RL functions internally, or to involve a third-party reverse logistics provider (3PRLP) to perform them.

The purpose of this research is to identify the suitability of the outsourcing option for a particular RL system, under a particular behavior for the return volume. To accomplish this goal, a complete analysis of the current existing RL systems in the U.S. is performed in Chapter 2, where the most important elements that determine the behavior of such systems are described. In Chapter 3, a quantitative analysis is performed by developing a Markov Decision Model (MDM), which allows us to model not only the return process,

but also the conditions under which a simple threshold policy is optimal. Such conditions are stated in terms of the cost parameters involved, as well as the return rate for the product considered.

The convenience of such kind of policy is supported by their ease in implementation, as well as their appeal to decision makers. In this case, the problem is reduced to identifying a threshold above which outsourcing is optimal, while to continue performing the Reverse Logistics internally is optimal below that threshold.

The hypothesis that outsourcing is a more suitable option for scenarios with greater variability in the return volume is also supported, both analytically and by studying a set of numerical examples, where it is shown how the threshold for outsourcing decreases while the probability of crossing any fixed threshold increases with the variability in the return volume. The development and solution of numerical examples is performed through a Matlab program developed for this instance. The states, rewards, and probabilities are computed, as well as the optimal action to take at each system state in each decision epoch, by solving the examples using backward induction through this program. These numerical examples demonstrate not only how the threshold is easily crossed when the variability on the return volume increases, but also when the length of the product's life cycle is shorter.

Finally, a set of conclusions are drawn, as well as the future research that can be developed based on this work.

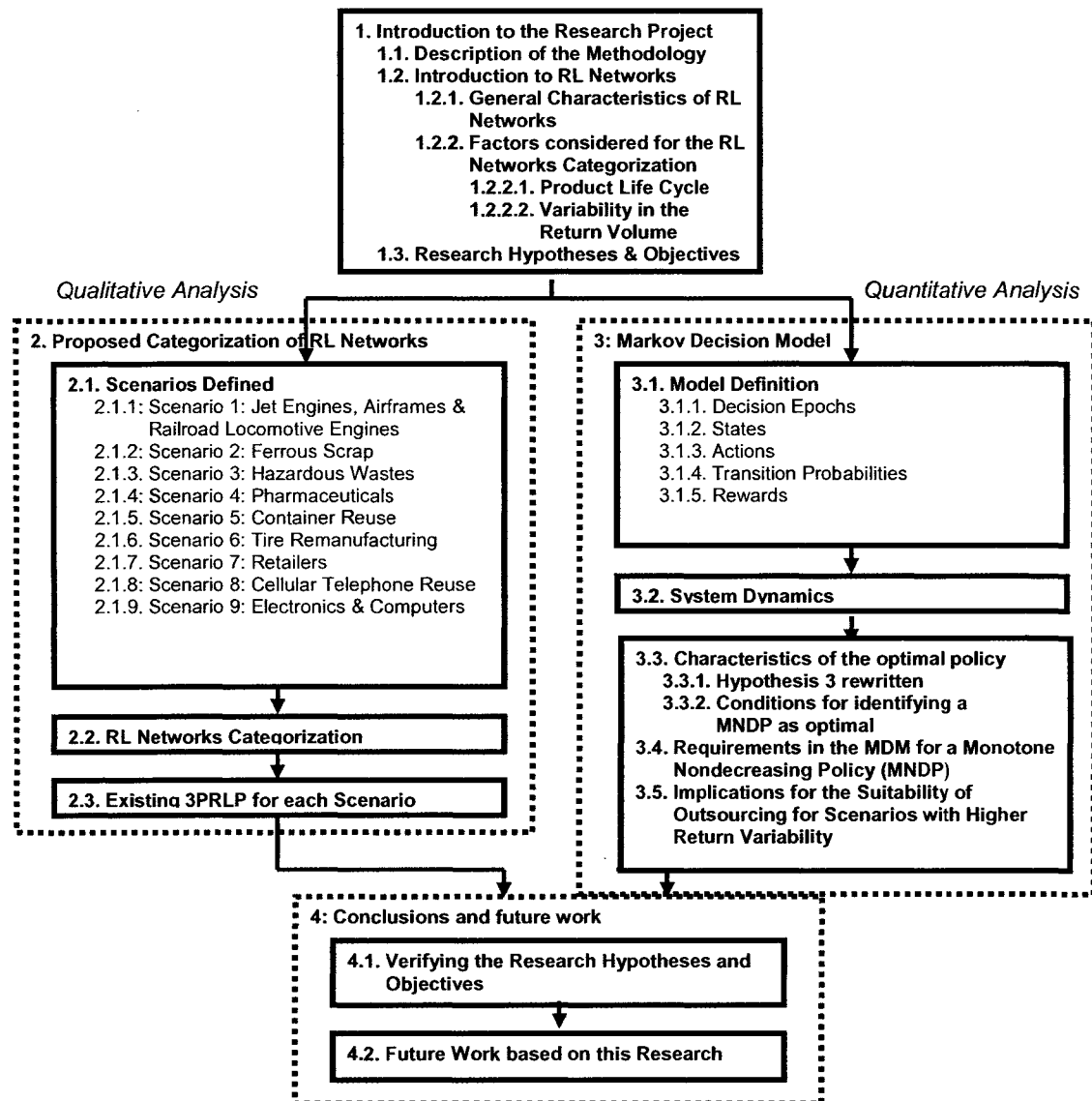
1.1. Description of the Methodology

This dissertation is developed as shown in Figure 1. As a first step, an introduction to the research project is given, which contains a description of the general characteristics of the RL networks, as well as the importance of two elements which are critical in every RL channel: the length of the product life cycle and the variability in the return volume. Once these two elements are described, the research hypotheses and objectives are

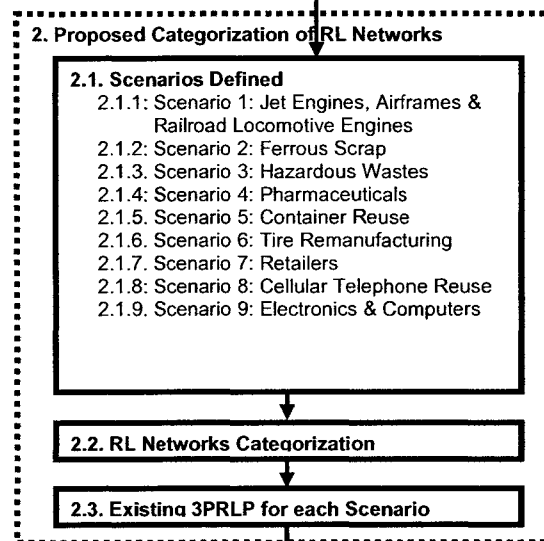
explained, which take us to the qualitative and quantitative analysis for RL systems to be developed.

The qualitative analysis corresponds to the description of some of the most important RL networks in the U.S. market, which are classified according to the two elements mentioned above. Also, several 3PRLPs that actually offer their services in each scenario are mentioned.

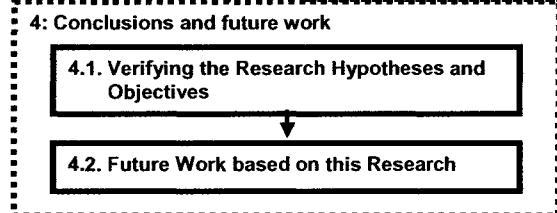
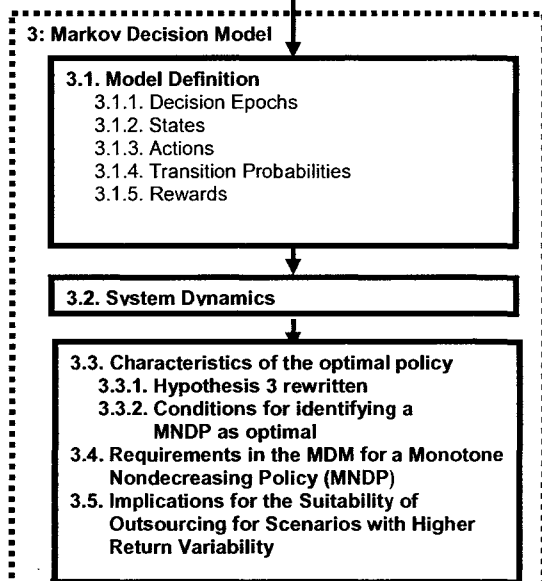
Figure 1. Stages developed in the Research.



Qualitative Analysis



Quantitative Analysis



On the other hand, the quantitative analysis corresponds to the development of a MDM, which represents the RL outsourcing problem faced by a firm. The decision epochs, states, actions, transition probabilities and rewards that compose such model are described, explaining also its system dynamics. With this information, one of the research hypotheses is rewritten, and the conditions for identifying an optimal monotone nondecreasing policy (MNDP) in this model are defined. The implications of such conditions on the return rate are discussed, and all the quantitative analysis is supported in a set of numerical examples.

Finally, a set of conclusions on the qualitative and quantitative analysis are developed, as well as the future work that can be developed based on this research.

1.2. Introduction to RL Networks

There are many reasons why products are returned, either by consumers or by the companies involved in the distribution chain. Retailers may return products because of damage in transit, expired date code, the model being discontinued or replaced, seasonality, excessive retailer inventories, retailer going out of business, etc. On the other hand, consumers can return products for such reasons as quality problems, failure to meet the consumer's needs, for remanufacturing, or for proper disposal.

Also, once products have reached the end of their useful life, they may be able to be remanufactured, refurbished or repaired; thus extending their life. These options can provide significant benefits in some instances, especially for products that have modular components (e.g. electronic equipment, computers) that can be replaced, upgraded and/or refurbished. The value of items that are remanufactured will typically be less than the same items produced for the first time. However, their value will be substantially higher than items being sold for scrap, salvage or recycling (Stock, 1998).

The importance of RL has increased in recent years. Currently, estimates of annual sales of remanufactured products exceed \$50 billion in the United States alone (Guide and van Wassenhove, 2003a). There are no worldwide estimates of the economic scope of reuse

activities, but the number of firms engaged in this sector is growing rapidly in response to the opportunities to create additional wealth, and in response to the growth in extended producer responsibility legislation in several countries. Unfortunately, even with this significant development for the RL market in recent years, not enough analytical models currently exist which assist in RL strategic decisions.

The RL systems classification that is initially proposed in the qualitative analysis of this document, is an attempt to develop particular decision-making tools according to the characteristics of the RL network under analysis. The planning, executing, controlling and optimizing activities performed in a RL system, or the decision to outsource these activities to a 3PRLP, will largely depend on the type of network that the firm is dealing with. For this reason it is relevant to initially structure and categorize the existing RL systems in the US market, as well as worldwide.

By identifying the particular characteristics that define each RL network, appropriate management tools and strategies can be developed. The first purpose of this research is to present a categorization that allows RL strategic decision-makers to identify the main differences between RL systems according to two relevant factors; to classify the previous research work developed in this field according to such categorization; and, as a first step in outsourcing decisions, to recognize if there are currently 3PRLP who provide services in each category.

Tibben-Lembke (2002) makes a clear explanation of the importance of considering the product life cycle to analyze RL systems, which is one of the factors considered in this categorization. This author explains the different behaviors that can be expected for the amount of returns, according to the length of the life cycle, depending on the type of product that the company is dealing with. However, he does not describe the expected length and behavior for the life cycle of particular products managed through specific RL systems. Thus, this research includes the life cycle as one of the two factors proposed to make this categorization.

On the other hand, Guide and van Wassenhove (2003a) state some relevant business aspects for several RL systems, describing the most important variables for them. But they do not state clearly the characteristics of the life cycle for the products managed on any RL system, which (as stated by Tibben-Lembke, 2002), is a critical issue to consider when analyzing any RL network. This is precisely one of the objectives of this research: to propose a RL systems categorization, based on the length of the lifecycle, as well as on the variability in the amount of returns. As it will be shown, by considering at least these two factors (which were not considered by these authors), any RL system can be clearly identified and classified, in order to determine the type of tools needed for its decision-making process.

Thus, the purpose of section 2, which corresponds to the qualitative analysis, is to present a new categorization of RL systems. The most important benefit of this categorization is that, by considering the life cycle and the variability in the amount of returns, the most important characteristics of any RL system can be stated. Even more, particular decision-tools can be developed according to such characteristics. In our case, the MDM developed in section 3 represents an outsourcing decision tool that can be applied to one of these particular scenarios, considering the variability in the return volume and the length of the product life cycle.

In this vein, it is relevant to identify that RL functions are often considered as non-core operations for most organizations, which are not always willing to perform them by themselves. These activities would just represent a “distraction” of the firm’s attention away from its core activities. Even more, as the basic economic justification for any form of outsourcing is the economy of scale associated with specialization (Daugherty and Drögue, 1997), this strategy is significantly relevant in RL programs. Then, a complete analysis of the partnerships or alliances in the current RL systems is necessary to achieve optimal results, and multiple organizations might be involved in the RL functions.

Therefore, it can be said that a critical issue in RL systems is whether or not to outsource these activities to a 3PRLP. Brito, Flapper and Dekker (2002) show some of the critical

success factors in RL, while Razzaque and Sheng (1998) develop a comprehensive literature in outsourcing logistics functions. On the other hand, Rabinovich, Windle, Dresner and Corsi (1999) make an examination of the current industry practices for outsourcing integrated logistics functions. But even though extensive research has been performed in relation to outsourcing logistics functions, not enough research has been made on the specific case of outsourcing RL activities. Furthermore, once the decision to outsource a defined set of RL functions has been taken, the selection of a 3PRLP is a critical issue to consider too. Only Meade and Sarkis (2002) develop a model for selecting and evaluating 3PRLPs, but this model assumes that the outsourcing option has already been taken by the firm.

In this research, the existing 3PRLPs for some of the scenarios to be described are mentioned, which allows us to identify not only the actors involved in the RL network, but also the complexity of that chain.

1.2.1. General characteristics of RL networks

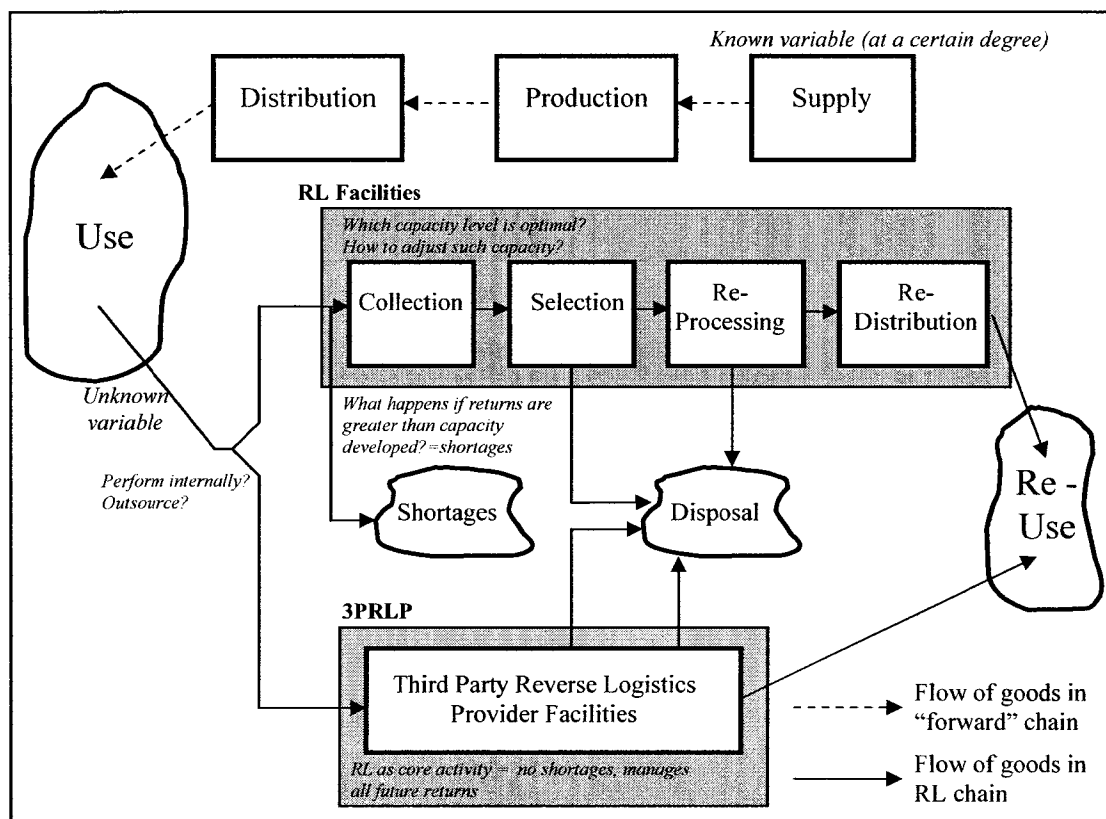
RL networks have several characteristics that differentiate them from the typical supply chain. First, RL networks encompass several supply chain stages. In this sense, RL fits well in the mindset of supply chain management, advocating coordination of the entire supply chain rather than considering single stages independently.

Roughly speaking, forward networks correspond to distribution networks encompassing supply, production and distribution stages (see Figure 2). The major differences between both contexts appear at the supply side. In traditional production-distribution systems, supply is typically an endogenous variable in the sense that timing, quantity and quality of delivered input can be controlled according to the system's needs. In contrast, supply is largely exogenously determined in RL chains and may be difficult to forecast. Hence, supply uncertainty is a major distinguishing factor between forward and RL networks.

In a typical forward chain, the demand for the good is uncertain, but the supply is not unknown (to a certain extent) and can be considered as a decision variable. As Kouvelis

(2001) states, this demand and supply uncertainty leads to alternate conclusions regarding the degree of outsourcing in both networks: usually, greater supply uncertainty increases the need for vertical integration in forward chains while greater demand uncertainty increases the reliance on outsourcing. However, this conclusion will always depend on the specific characteristics of the network that is being analyzed, as well as the fact that RL does not represent a core activity for the firm, while it does represent the core activity for the 3PRLP, which may have enough capacity as a consequence of focusing on managing returns.

Figure 2. The RL chain



Forward networks typically do not include an “inspection” stage similar to RL networks. Destinations of goods flows are, in general, known beforehand with more certainty as compared to the quality dependent processing routes in RL chains. While there may be some particular exceptions, this is not the major focus of traditional forward networks. Therefore, network structures may be more complex in RL, including more interdependencies. Another element that may lend RL networks a higher complexity is

the potential interaction between collection and redistribution, e.g., combined transportation in closed-loop networks. However, this network complexity depends on the specific recovery process and may vary considerably.

Another fundamental difference between forward and RL networks is identified in the number of sources, which tends to be fairly large in RL as compared to the number of supply points in a traditional setting. Bringing together a high number of low volume flows therefore appears to be characteristic of RL networks in particular.

However, both networks can also be analyzed together. As Guide and van Wassenhove (2003a) state, closed-loop supply chains (which are composed of the typical forward-supply network and the RL network) can be viewed as a business proposition where profit maximization is the objective. The characteristics of such maximization will depend on the forward supply chain characteristics, as well as the RL network composition, which will be defined and classified in this document.

1.2.2. Factors considered for the RL networks categorization

The RL networks categorization shown in Chapter 2, which is considered as the basis for the MDM developed in Chapter 3, is based on two factors that determine the structure and characteristics of every RL system. These factors are the length of the product's life cycle and the variability in the return volume during the life cycle. The reasons to consider these elements will be explained in this section, while sections 1.2.2.1 and 1.2.2.2 will describe each one of them in detail.

The length of the product life cycle varies across products and industries (Rogers and Tibben-Lembke, 1999). Since it is not easy to identify where a real product is in the life cycle once it moves past the introductory and growth stages, every firm must look for demand turning points. These can be seen if the company understands past history and the marketplace, and will allow the firm to understand the expected behavior for the volume of units returned through its RL system.

One of the most important difficulties for every firm, when analyzing the life cycle of its products, is to admit that it is at the end of its life cycle. However, if this challenge is faced adequately, product life cycle analysis can become a critical piece for an adequate RL system management. As it will be explained in the next section, the stage where a product is located in its life cycle is significantly related to the amount of units returned through its RL network.

Competitive environments have caused the product life cycle for many consumer goods to continually shrink (Guide and van Wassenhove, 2003a). As an example, many consumer electronics, such as mobile telephones, have less than six months between new model introductions. Products such as these that have a very short shelf-life and that can be restocked without furthering handling may best be returned to the originating distribution center (Gooley, 2003). One example is catalogue sales, where items that come back unopened can almost immediately be returned to inventory and become available for sale. This situation significantly facilitates the RL management. However, this is not the case for all types of products.

The management of the product returns process in a timely and effective manner in the case of short life cycle goods presents enormous difficulties compared to products whose life cycle length is longer. But it is relevant to look not only at the length of the product's life cycle, which affects the amount of returns in each period, but also at the variability in such volume from one time period to the next. Characterizing products according to average amounts of returns is not sufficient since the variability of the return volume will also affect the structure and configuration of the RL system developed to deal with them.

1.2.2.1. Product life cycle

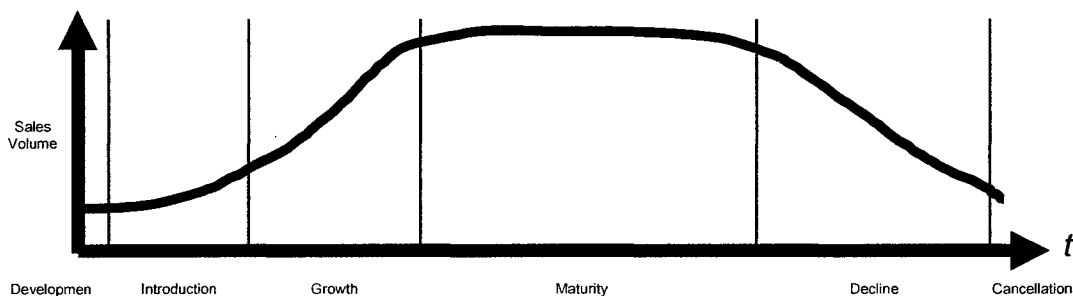
Not all products are fortunate enough to have periods of significant growth and stability. Many products either fail to have any significant sales, or have short sales lives. If the product has a very short life, the retailers may return large volumes of unsold product to the manufacturer.

A typical example for this type of behavior is the computer market. In this sector, the introduction of new components accelerates the demise of computer models previously introduced, as the manufacturer must introduce new models (just as its competitors are doing) that will reduce the sales of the existing models (Tibben-Lembke, 2002).

In order to understand the RL flow behavior, it is relevant to look at the product's life cycle. Tibben-Lembke (2002) identifies six phases that are defined during the life cycle of any product: development, introduction, growth, maturity, decline and cancellation. Figure 3 shows the expected sales volume during these stages.

The amounts of units returned during each one of these stages differ significantly. The major issues that define the volume of the units returned through the RL system for a product model (such as a specific model number of a particular product) during these six phases are:

Figure 3. Stages of product life cycle.



Development phase.

When a new model of an existing product is being developed, few challenges are to be expected in the development phase. Because the new product has minor changes compared with the old, clients that buy the current product are likely to be interested in the revised product, and the RL policies and procedures for dealing with the old product are likely to work satisfactorily with the new.

Introduction phase.

Early in the introduction stage, the firm can begin making plans for dealing with the products which eventually will be returned. As with a new form of an established product, clients will be familiar with the product, and be able to estimate the demand in the secondary market. Lee and Whang (2002) describe clearly the impact of the secondary market on the entire supply chain.

During the introduction stage, the company must also begin dealing with the flow of returned product. Because (in most cases) a new model is a minor modification of the existing product, production difficulties in adapting to the new model should be minimal. The minor modification also means demand for the new model would be expected to be very similar to demand for the previous model. In the case of a new model of a popular product, sales may be high from the beginning or start small and grow quickly, as customer demand for an established, known product is transferred to the new product. In these cases, Tibben-Lembke (2002) suggests that the product will skip the introduction phase.

Growth phase.

Increasing sales of a new model are unlikely to lead to production difficulties. During this phase, returns volume will substantially increase, as sales increase, although the rate of returns (as a function of the sales volume) may be unchanged. However, as more customers are attracted to the product, these new customers may be less knowledgeable about the product, and the rate of “non-defective defectives” may increase. In the same vein, the variability in the rate of returns between consecutive periods is expected to increase.

Maturity phase.

As sales for the model reach its maturity stage, the amount of returns will also be expected to reach a stability period. Given a relatively constant amount of units sold per period, the volume of returns will also be expected to reach such stability. However, it is important to note that the volume of returns in a particular period is related not with the volume of sales in the same period, but with the historical sales in the previous periods.

Though the firm might reach this stability in its sales volume, the variability in the rate of returns between consecutive periods may increase.

Decline phase.

In forward distribution, during this phase, the company is trying to determine how long it can continue to sell the product profitably before it needs to terminate it. In RL, the company does not directly decide when to stop accepting returns. Rather, the last date for allowing returns of a product will depend on the company's returns policy and the date of the last sale of the product. If, for example, customers can only return a product for 90 days after the last sale, then returns may come to the retailer as long as 90 days after the last sale.

As sales of the product fall, its price on the secondary market also is likely to fall. However, if the model sales are declining because a newer, similar model has been introduced, secondary market firms will be very interested in purchasing the product. Because this product is similar to its newer replacements, value retailers will be eager to be able to sell a model that is not very different from the newest models.

Cancellation phase.

When a product reaches the end of its life, the volume of customer returns will continue to decrease before stopping altogether. Even if the product has sold well, at the end of its life, retailers may send any unsold product back.

Despite the fact that sales of this model are falling, sales of similar, but newer models will unlikely continue to be strong. Therefore, the secondary market demand for the product will remain strong. This implies that the secondary market demand for the product will remain strong. Some other firms might be interested in buying up all remaining product at the end of the product's life, although vendor restrictions about product placement will remain high.

Conclusions about the impact of the product life cycle on the RL flow.

As it was explained in this section, the product life cycle strongly determines the expected amount of returns for a particular product over time. However, the characteristics of such returns will also depend on the length of the life cycle (not all products or industrial sectors have similar length for its product's life cycles), as well as the particular characteristics of that product.

1.2.2.2. Variability in the return volume

The volume change in supply is much greater in RL channels due to the many uncertainties associated with product and material life return rates. To cover the different amounts of variation faced for different products, particular return volumes need to be considered during the life cycle. For example, commercial returns from retail and Internet-based sales are a concern in North America and a growing concern in Western Europe. In 2001, the cost of returns for Internet sales was averaging twice the value of the product (Guide and van Wassenhove, 2003b).

Though different products may have equal (average) return volumes for each life cycle stage, the variability about that average during the entire life cycle can be significantly different. Higher variability complicates the management process for these returns.

A significant return volume is needed to justify the considerable costs of establishing a separate RL system, including the expense of a building, materials handling systems, information system and a large workforce. However, the variability in this return volume is also a significant factor to consider when making strategic decisions in any RL system.

An initial argument for relating the variability in the return volume to the outsourcing decision can be stated as follows: due to extremely high variability in such returns, it may not be economically feasible for a firm to develop its own RL facilities to deal with that flow, given that the amount of units to be returned will be uncertain over time, and the required capacity will be changing constantly. This may be effectively accomplished by involving a 3PRLP, which specializes in these activities, and can take advantage of

the economies of scale to convert RL functions in a profit-creating activity into the closed-loop chain.

On the other hand, if there is a relatively low variability in the expected amount of returned units, these firms may be able to implement their own RL systems without a particular need for another party involved. However, this situation will be closely related to the length of the life cycle, which determines the need for fast, but adequate decisions about such RL systems.

Then, though the variability under analysis may vary over the life cycle, for simplicity we will assume that it is constant and categorize it as a low, medium or high change level throughout the product's life.

1.3. Research Hypotheses & Objectives.

Considering the characteristics for the RL networks defined before, as well as the elements to be considered for its categorization, the hypothesis to be verified in this research can be stated as follows:

Hypothesis 1:

The existing RL Networks can be logically categorized according to the length of the product's life cycle and the variability in its return volume.

Hypothesis 2:

Some of the most important 3PRLPs offer their services in RL channels that manage products with a relatively short life cycle, and high variability in its return volume.

Hypothesis 3:

Outsourcing is more likely to be optimal for returns of products that have short life cycles and high variability.

Considering the previous hypotheses, the objectives of this research can be defined as:

Research Objective 1:

Propose a categorization of RL Networks, according to two critical factors: the length of the product life cycle, and the variability in the return volume.

Research Objective 2:

Identify the scenarios (according to the proposed categorization) where most 3PRLPs currently offer their services.

Research Objective 3:

Formulate a Markov decision model and establish conditions under which a monotone policy is optimal.

Research Objective 4:

Analytically prove that the threshold that determines the suitability of the outsourcing option is easily crossed in scenarios where the RL system corresponds to products with a shorter life cycle, and relatively high variability in its return volume.

As it is mentioned in the previous hypotheses and objectives, the purpose of this research is to initially categorize the existing RL systems according to the variability in the return volume, under a short, medium or long life cycle. After this, the next step is to quantitatively verify the optimality of an outsourcing strategy for a RL system, which will be accomplished by developing a MDM that allows us to represent the characteristics that define a particular scenario. Then, by considering the length of the product life cycle and the variability in its return volume, a particular firm will be able to use the proposed model to determine its optimal strategy; i.e., either to perform the RL activities internally or to follow an outsourcing strategy.

2. Proposed categorization of RL networks

Having defined the two factors for the proposed categorization, the RL networks are classified in the next scenarios.

2.1. Scenarios defined.

2.1.1. Jet Engines, Airframes and Railroad Locomotive Engines.

These types of products are the first ones to be described, because their corresponding RL systems are the easiest to structure and manage. Even though all of these products have a complex nature and physical size that makes testing and remanufacturing operations very difficult, their RL network is relatively simple.

It is important to state that the volume of these products has a significant impact on transportation costs. Shipping many small lots of returned goods over long distances to and from a centralized facility can be expensive. Typically, the life cycle for these types of products is significantly long, because their corresponding markets do not really demand new models in short periods of time. On the other hand, the variability in the return volume for these products are often very low, with each product being an essentially new project to plan. For example, the US Navy required over three years to completely overhaul the carrier the USS *Enterprise* (Guide and van Wassenhove, 2003a). Then, it can be concluded that all of these products have a relatively long life cycle, and the variability in their volume of returns is low.

2.1.2. Ferrous Scrap in the Steel Industry.

The RL systems developed for the steel industry represent a significant volume considering the characteristics of this industrial sector. An estimated of 50 million tons of ferrous scrap is managed each year in North America through these RL networks (Johnson, 1998). The ferrous scrap recycling system represents a significant level of economic activity, with estimated revenues of \$8 billion in the United States alone.

Similar to the characteristics of the RL systems for airframes, jet engines and railroad locomotive engines, the life cycle for this sector is considered to be long. Though the steel industry technology has changed in the last years, the life cycle for the materials managed through these return channels is still considered to be long.

Several efforts have been performed to minimize the amount of scrap generated, but these efforts do not directly affect the variability in the ferrous scrap volumes put into these channels, which are considered to be low. These efforts only affect the average amount of units managed, which is expected to decrease through time. The effort in minimizing this volume of scrap is related to the high transportation and disposal costs for these RL systems.

2.1.3. Hazardous Wastes.

Hazardous waste RL systems are helpful for solving waste-induced environmental pollution problems that accompany high-technology industrial development (Hu, Sheu and Huang, 2002). Given the particular characteristics of the products managed in this type of RL networks, it is difficult to coordinate all the activities involved in them (collection, storage, distribution, transportation, disposal, etc).

As the model proposed by Hu, Sheu and Huang (2002) shows, it makes sense to consider the variability in the return volume as relatively low. In practice, these time-varying demands can be measured readily from order entries of the waste-treatment company.

Also, given that the product life cycle in this particular sector will largely depend on the type of technology used by the company (which generates the wastes to be managed), its length is considered to be relatively long, because every investment in new technology typically represents a significant amount of money and resources for a firm. This causes most companies to acquire new technology on a long-range basis, which causes the

amount of hazardous wastes to be fairly stable. Then, the cycle length for this RL network is defined as medium.

However, it is also important to mention that, as Stock (1998) states, “the best way to reduce waste is not to create it”. This principle is the main cause of the efforts in this type of RL to reduce the volumes managed through these systems.

2.1.4. Pharmaceuticals.

As stated in the previous sectors, the specific needs of the industry to which the company belongs also influences the choice of the configuration for the RL system, as well as the convenient facilities. Because the pharmaceutical industries affect consumer health and safety, these firms must segregate return goods to prevent them from mingling with or contaminating new merchandise (Gooley, 2003). Using separate returns processing centers guarantees segregation. It also facilitates physical handling procedures and records keeping that are required by federal regulations in certain industries.

The average life cycle length for these products is considered to be medium. Even though new products are put into the market as a consequence of the medical advances and research in this field, existing products stay in the market for a considerable time period. But on the other hand, most of these products have a date of lapsing, which causes some returns that (by government regulations) must be managed adequately by every firm.

The variability in the volume in the pharmaceutical industry are relatively low, because firms know (to a certain extent and in most cases) the expected demand for their products, which helps them in forecasting sales. Also, returns because of lapsed products or defective factors are relatively medium.

2.1.5. Container Reuse.

Historically, container remanufacturing may be one of the oldest forms of product reuse (Guide and van Wassenhove, 2003a). In the past, drink bottles were regularly refilled after being acquired from the consumer. Product acquisition was done directly from the consumer, e.g., milk bottles, or at resellers (soft drink bottles) who participated in a deposit system to encourage returns. While bottle refilling is not commonly practiced in the United States anymore, there are several countries in Latin America that still practice this.

Given the actual conditions in the markets where container remanufacturing is still practiced, it can be observed that the life cycle for these items is long, because (as a consequence of the market demand for bottle refilling), there have not been significant changes that might cause the incorporation of different drink bottles.

Given the fact that all of the products consumed in this sector generate an item to be managed through the RL system, the amount of units returned is highly related with the sales volume. This volume is considered to be stable for this sector, which implies that the change in such amounts of returns is considered to be medium. As Guide and van Wassenhove (2003a) note, toner cartridge recycling and single-use camera remanufacturing can be seen as contemporary instances of container reuse.

2.1.6. Tire Remanufacturing.

Tire remanufacturing has enjoyed periods of popularity during times of economic crisis or during wartimes when rationing has been in effect. The European Union recently passed legislation requiring extended producer responsibility for tire manufacturers. In order to comply with this new legislation, tire manufacturers will have to arrange for economic end-of-life disposition for all their products. On the other hand, tires retreaded for commercial trucking applications have a ready market since tires are often one of the largest expenses for trucking fleet owners. The lower cost of remanufactured tires makes them attractive for fleet managers.

The RL chain for tire retreading has some elements in common with industrial remanufacturing (Guide and van Wassenhove, 2003a). However, the volume of tires in use is enormous. Additionally, tires are bulky and expensive to transport, and the residual value remaining may be low, especially when compared to the cost of new replacements. Tire remanufacturing is rarely profitable for passenger tires, but financially attractive for commercial tires.

These characteristics for the tire retreading market, as well as the introduction of new models, classify the life cycle for these products as medium. The models developed stay in the market for a certain amount of time, and the variability in the returns are also considered to be medium. This variability is a consequence of tire sales, as well as the average use of such tires, which is (to a certain extent) adequately estimated by the manufacturers of these products.

2.1.7. Retailers.

Where to send an item that has been returned, or how to dispose of the item, is one of the most important decisions to be made in retail RL. Although case studies have been written in the end-of-product-life decision making, there is still a significant opportunity area for RL systems in this sector.

A returned product that cannot be sold as new will typically be sold for a fraction of its original cost. Choosing the right disposition option can mean a revenue increase of a number of percentage points, and can make a significant impact on the corporate bottom line. For example, by improving disposition decision making, some large retailers have realized savings of as much as \$6 million per \$1 billion in retail sales (Jedd, 2000).

A clear example of this situation can be seen in J.C. Penney's multi-channel return system (J.C. Penney, 2003). By being a catalog and direct retailer, J.C. Penney deals with very high return rates of more than 35%, the mean being 25%. Because return rates for many of the catalog retailers have traditionally been very high, a reduction in both

the number of returns and the cost of those returns has been desirable, but not accomplished to date.

In this context, insufficient attention paid to the RL problem can lead to significant financial problems for retailers. For firms that have not optimized the returns process, the cost of returning products can be as much as 70% higher than the cost of the initial shipment due to unpredictability of return volume and frequency.

On the flip side, the growing wave of product returns is creating a boom for online auction, liquidation, and disposition companies like Overstock.com, eBay, Amazon.com, and others. These firms receive a commission for selling other parties' inventory on their Web sites. The business goal for these firms is to solve a significant pain point for manufacturers by drastically cutting the costs of handling product returns, damaged products, and overstock mistakes.

Considering the characteristics of the products managed in these RL networks, the average life cycle for them is considered to be medium. On the other hand (as described in the previous examples) the variability in their amount of returns is considered high, due to the changing amounts of product returns registered for this sector.

2.1.8. Cellular Telephone Reuse.

The cellular communications industry is a highly dynamic market where the demand for telephones changes daily. Demand may be influenced by the introduction of new technology, price changes in cellular airtime, promotional campaigns, the opening of new markets, churn (customers leaving present airtime providers), and the number of new cellular telephones manufactured. Additionally, there is no worldwide standard technology, and this necessitates dealing in a number of often disparate technologies and standards. These global technology differences make regional remanufacturing activities difficult since there may be no local market for certain types/models of phones, requiring a firm to manage global sales and procurement. Also, cellular airtime providers may

limit the number of telephones supported by their system, and the dropping of a phone model by a major carrier can greatly affect a local market.

A clear example for this sector is ReCellular, Inc. (Guide and van Wassenhove 2003a). This firm refurbishes cellular phones when necessary to add value for existing orders, and buys and sells wireless handsets of all technologies. The company offers remanufactured (refurbished) products as a high quality, cost effective alternative to new cellular handsets. Customer services include: grading and sorting, remanufacturing, repackaging, logistics, and trading and product outsourcing (all services are specific to cellular handsets and accessories). ReCellular operates globally with a presence in South America, the Far East, Western Europe, Africa, the Middle East and North America. The company has also plans to expand operations to provide better coverage throughout the world.

Due to the changing characteristics of the models and handsets constantly put into the market, the life cycle for this type of products is significantly short. In the same vein, due to the high number of models and companies into this market, as well as the changing conditions in the service offered to the users, the variability in the amount of returns are considered as high.

2.1.9. Electronics and Computers.

As Rogers and Tibben-Lembke (1999) note: “we are in an industry with 60-day product life cycles and 90-day warranties”. In the actual market conditions for the electronics and computers market, customers currently bring products back to a high extent. The life cycle of a computer or other electronic product is extremely short when compared to other consumer durable goods. As these authors also state, returns in this industrial sector can lower profits by as much as 25 percent, which makes RL a serious business.

The electronics and computers RL systems may hold one of the most important promises due to the volume of product available to reuse. But at the same time, these types of RL

networks represent some of the greatest challenges due to its complexity in time and variability in the rate of return.

Product acquisition is very difficult. These types of products are used globally, but the rate of technical diffusion is different in various geographic areas. This requires that a successful operation will have worldwide collection and distribution markets, and these markets will not be in the same geographic areas. Supply and demand rates and prices are extremely volatile. The products are also perishable items since the value of a remanufactured item may drop daily because of the rapid rate of technology progress and the rate of technology diffusion. There are also multiple options for reuse since products may be sold in graded as-is condition or remanufactured. Each option has a different selling price, which is quite dynamic.

Then, it is clear that the electronics and computers sector manage the products with shorter life cycles, and (as a consequence of the changing conditions in these markets) the variability in the amount of returns are extremely high.

2.2. RL networks categorization.

Once the previous scenarios have been defined, the categorization matrix shown in Table 1 can be constructed according to this analysis.

As it can be identified in Table 1, three categories have been defined for each factor: short (1 to 12 months), medium (12 to 36 months) and long (above 36 months) product life cycles, and low, medium and extremely high variability in the amount of returns. This categorization implies nine possible categories, where six of them have been identified for the most important RL networks currently in existence.

This categorization allows us to identify the causes for different management practices in each scenario. The relationship between the life cycle length and the variability in the amount of returns determine, to a great extent, the RL network configuration, stages and parties involved.

Table 1. Scenario Matrix

Product Life Cycle Length	Variability in the Return Volume		
	Low	Medium	Extremely High
Long	<i>Scenario 1:</i> <i>Jet engines, Airframes & Railroad Locomotive engines</i> <i>Ferrous Scrap in Steel Industry</i>	<i>Scenario 3:</i> <i>Container Re-Use</i> <i>Copy/Print Cartridge</i> <i>Single-Use Cameras</i>	
Medium	<i>Scenario 2:</i> <i>Hazardous Wastes</i> <i>Pharmaceuticals</i>	<i>Scenario 4:</i> <i>Tires</i>	<i>Scenario 5:</i> <i>Retailers</i>
Short			<i>Scenario 6:</i> <i>Cellular Telephone Reuse.</i> <i>Electronics & Computers</i>

According to this categorization, some research works performed in each scenario are identified in Table 2.

Table 2. Research works performed on each scenario.

Scenario	Products	Authors
1	<i>Jet engines, airframes and railroad locomotive engines.</i>	<i>Guide and Wassenhove (2003a)</i>
	<i>Ferrous Scrap</i>	<i>Johnson (1998)</i>
2	<i>Hazardous wastes</i> <i>Pharmaceuticals</i>	<i>Hu, Sheu and Huang (2002)</i>
		<i>Stock (1998)</i>
		<i>Gooley (2003)</i>
		<i>Teunter, et. al. (2000)</i>
3	<i>Container remanufacturing.</i>	<i>Guide and Wassenhove (2003a)</i>
		<i>Goh and Varaprasad (1986)</i>
		<i>Kelle and Silver (1989)</i>
4	<i>Tires</i>	<i>Rogers and Tibben-Lembke (1999)</i>
		<i>Guide and Wassenhove (2003a)</i>
	<i>Copy and Print Cartridge</i>	<i>Guide and Wassenhove (2003a)</i>
		<i>Krikke, et. al. (1999)</i>
		<i>Rogers and Tibben-Lemke (1999)</i>
<i>Single-use cameras</i>	<i>Xerox (1999)</i>	
	<i>Goldstein (1994)</i> <i>Guide and Wassenhove (2003a), Kodak (1999)</i>	

Table 2. (Continued).

Scenario	Products	Authors
5	<i>Retailers</i>	<i>J.C. Penney (2003)</i> <i>Jedd (2000)</i> <i>Gooley (1998)</i> <i>Hamilton (2001)</i> <i>Hoffman (1998)</i> <i>Rogers and Tibben-Lembke (1999)</i> <i>Winter (2001)</i>
6	<i>Cellular telephones</i>	<i>Guide and Wassenhove (2003a)</i>
	<i>Electronics and Computers</i>	<i>Autry, Daugherty and Richey (2000)</i> <i>Knemweyer, Ponzurick and Logar (2002)</i> <i>Moyer and Gupta (1997)</i> <i>Rogers and Tibben-Lembke (1999)</i> <i>Veerakamolmal and Gupta (1998)</i>

Table 3: Complexity of the management and decision-making process for each scenario.

Scenario:	Products identified:	Complexity of its management process:
1	<i>Jet engines, Airframes & Railroad Locomotive engines</i> <i>Ferrous Scrap in Steel Industry</i>	<i>Relatively Low</i>
2	<i>Hazardous Wastes</i> <i>Pharmaceuticals</i>	<i>Medium</i>
3	<i>Container Remanufacturing</i>	<i>Relatively Low</i>
4	<i>Tires</i> <i>Copy/Print Cartridge</i> <i>Single-Use Cameras</i>	<i>Medium</i>
5	<i>Retailers</i>	<i>Relatively High</i>
6	<i>Cellular Telephone Reuse.</i> <i>Electronics & Computers</i>	<i>Extremely High</i>

Even though there are several challenges in the management process for each scenario, the complexity for this process is higher in some of them, due to the short length of the life cycle, which forces the firm to take fast, but at the same time adequate decisions, as well as a considerably high variability in the return volume, which increase the uncertainty about the volume of units put into the corresponding RL system. Table 3

summarizes the complexity for the RL management process and strategic decision-making for each scenario.

2.3. Existing Third-Party Reverse Logistics Providers (3PRLP) for each scenario.

When analyzing outsourcing decisions for RL, the fundamental factor to consider is whether there is a viable 3PRLP for the type of RL network required. Below we identify the existing 3PRLP in each one of the scenarios described above.

Outsourcing to a 3PRLP has been identified as one of the most important management strategies for RL networks in the recent years. In this vein, Meade and Sarkis (2002) state three different choices that can be made with respect to the development of any RL function: to do nothing, to develop an internal RL function, or to find a 3PRLP and partner with them. Krumwiede and Sheu (2002) show a particular model for RL entry by 3PRLP, which helps those companies who would like to pursue RL as a new market. Also, Meade and Sarkis (2002) develop a model for selecting and evaluating 3PRLP. However, this model does not represent a tool for determining whether or not to outsource RL activities, but it helps in the decision of selecting a 3PRLP once the outsourcing strategy has been chosen by the firm.

Even though there are several 3PRLPs in some of the scenarios described, one of the most important issues in RL systems is that some of them (that are currently desiring to enter the RL service market) are not really prepared to effectively address these service needs due to the lack of knowledge of RL networks (Dowlatsahi, 2000).

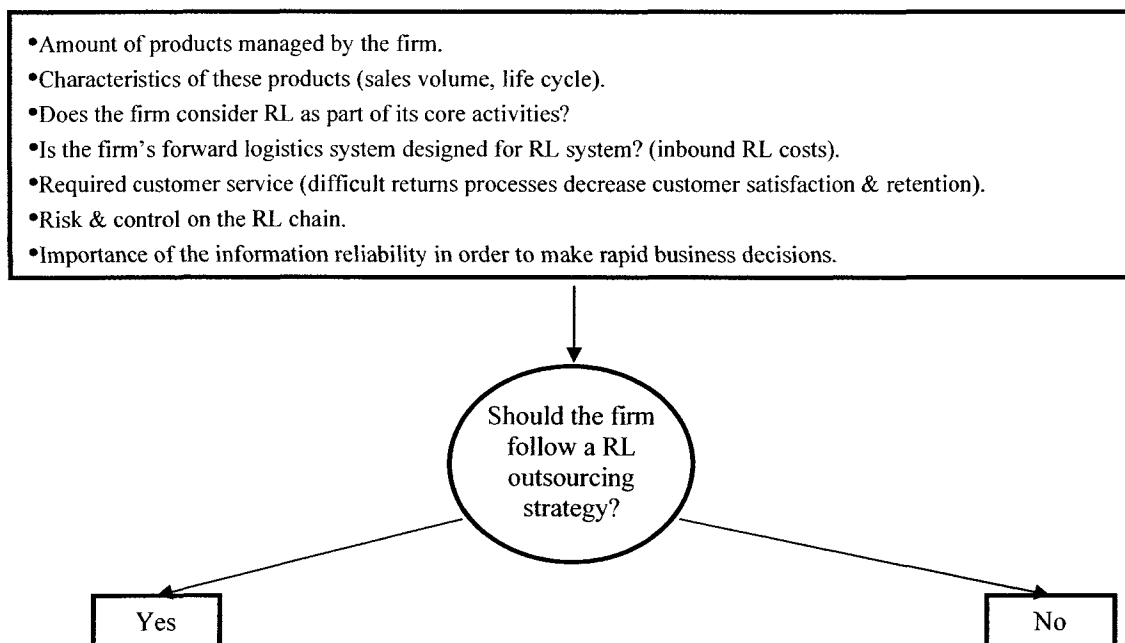
The decision on whether or not to outsource depends on several elements. Rao and Young (1994) explain the critical factors that influence the outsourcing decision for logistics functions. However, the particular factors to be considered in RL systems are graphically described in Figure 4.

As it can be identified in this figure, different elements need to be considered when an outsourcing strategy is going to be taken for a RL system. One of the most important issues is to define if the firm considers RL activities as part of its core functions. When

this is not the case, outsourcing might represent a good alternative in order to allow the firm to “focus” on its core activities.

Also, the cost of managing a returned item is one of the most important factors when choosing how to dispose of it, as well as the price to be received for it, if such a price exists. As stated in the previous sections of this document, these factors will differ according to the scenario where the RL system can be classified. The relative importance of these elements varies between companies, depending on their sizes, characteristics, products manufactured, managerial strategies and goals, etc. The amount of money invested in these activities will be a critical issue too.

Figure 4. Factors to consider when following an outsourcing strategy for a RL system.



Considering the proposed categorization of RL systems, some existing 3PRLPs for each scenario are identified in Table 4.

As it can be identified in this table, some of the most important 3PRLPs are located in scenarios 6 and 7. The reason behind this situation is precisely identified by the characterization stated in this document. Due to the extremely high variability in the rate of returns for the products managed in these RL systems, it is not economically feasible

at all for a firm to develop its own RL facilities to deal with that flow, given that the amount of units to be returned will be significantly uncertain over time, and the required capacity will be changing constantly. The complexity of this situation increases when the life cycle for this type of products is extremely short, which requires quick but adequately decisions for these RL systems, in order to efficiently respond to such changing conditions. This can effectively be accomplished by involving a 3PRLP, which specializes in these activities, and can take advantage of the economies of scale to convert RL functions in a profit-creating activity into the closed-loop chain.

Table 4. Existing 3PRLP for some scenarios.

Scenario:	Products identified:	Existing 3PRLP:
2	<i>Pharmaceuticals</i>	<i>USF Processors</i>
3	<i>Container Re-Use</i> <i>Copy/Print Cartridge</i> <i>Single-Use Cameras</i>	<i>GATX Logistics</i> <i>Burnham</i>
5	<i>Retailers</i>	<i>GENCO Distribution System</i> <i>Service Merchandise</i> <i>Redwood Systems</i> <i>Prime Logistics</i> <i>Menlo Logistics</i>
6	<i>Cellular Telephone Reuse.</i> <i>Electronics & Computers</i>	<i>ReCellular, Inc.</i> <i>SSI Supply-Chain Services</i> <i>GATX Logistics</i> <i>Burnham</i> <i>InSite Logistics</i> <i>SSI Supply-Chain Services</i>

On the other hand, the rest of the scenarios do not present several 3PRLPs for their RL systems, given that the life cycle for its products and the variability on its returns allow the firm (in most cases and at a certain extent) to develop its own facilities to deal with this flow, even RL may not be part of its core activities. The relatively low uncertainty on the amount of returns, and the longer time periods for planning, developing and implementing RL systems, allow these firms to implement their own RL systems without a particular need for another party involved.

The qualitative analysis in this section helps us answer hypotheses 1 and 2, as well as accomplish research objectives 1 and 2. Based on this information, a quantitative analysis will be developed in the next section. Such analysis will be performed by developing a Markov Decision Model that will consider the two elements mentioned in the RL networks categorization proposed: the length of the product life cycle and the variability in its return volume. By considering these elements, the suitability of the outsourcing option will be evaluated, which will help us in our effort to answer the third research hypothesis as well as accomplish the rest of the research objectives.

3. Markov Decision Model.

The MDM to be proposed represents the uncertainty in the return volume for a particular scenario, as well as the convenience of the outsourcing option implied for it. To accomplish this, it is relevant to identify that in most cases, any return volume is nothing but a consequence of the amount of units historically sold by a firm, given that a fraction of them is returned through its RL system. Considering this situation, the MDM developed will assume a particular sales function and maximum sales level as known, which is related to the scenario under analysis. These elements can be defined according to the historical data related to this scenario, and, by including a particular return rate (the fraction of units sold that can be expected to be returned), the variability in the amount of returns for each period during the entire planning horizon, which includes the product life cycle, can be determined.

Considering this, the following notation is defined for the MDM proposed:

t = Decision epoch.

L = Length of the product life cycle, which will depend on the particular RL scenario considered. Recall from the qualitative analysis performed in section 2, that such length strongly determines the characteristics of any RL system.

W = Time length defined by the firm to continue managing the returns for the product analyzed, after the last sale was made.

T = Length of the horizon analysis, $T=L+W$.

M =Maximum sales level expected.

r = Return Rate. This value represents the expected fraction of sold units to be returned in the next period, according to the RL scenario considered. Recall from section 2 that the variability experienced in the return volume affects the characteristics and behavior of any RL system. In this MDM, such volume is determined by the amount of sold units outstanding in the market, given that a fraction of them (represented by r) are expected to be returned. Then such value, as well as the particular sales function defined and maximum sales level expected (M), will determine the expected return volume, as well as the variability in such volume during any period t , which is

one of the two criteria considered for the RL networks categorization previously proposed.

k_t = RL capacity defined by the firm at the beginning of period t .

x_t = Amount of units returned in period t , which as mentioned before, is nothing but a consequence of the sales function and return rate r defined for the scenario considered.

s_t = Amount of units sold by the firm during period t .

S_t = Cumulative sales experienced by the firm from period 1 to the end of period t .

w_t = Cumulative amount of units returned from period 1 to the end of period t .

n_t = Amount of units outstanding in the market at the end of period t .

Also, the next assumptions are implied in the MDM to be developed:

Assumption 1:

It is assumed that a particular sales function and a maximum sales level M can be defined, which determine the value for s_t in each time period. Also, given that s_t determines the value for S_t as follows:

$$S_t = \sum_{i=1}^t s_i \quad (1)$$

Then such sales function and M value imply that both s_t and S_t can be estimated in advance. These elements are defined according to the particular scenario under analysis, and can be determined according to historical data related to it.

Assumption 2:

As Toktay (2003) states, the number of periods between when a product is sold and when it is returned can be modeled as a geometric distribution. From this, it follows that knowing n_t at time t , the number of returns in period $t+1$ has a binomial distribution with parameters n_t and r , such that the expected amount of returns in the next period can be obtained as:

$$E(x_{t+1}) = n_t r \quad (2)$$

Where as mentioned before, n_t corresponds to the number of products outstanding at the end of period t ; i.e., products that were sold during the first t periods, but that have not been yet returned:

$$n_t = S_t - w_t \quad (3)$$

Then, the function for the expected amount of returns implies that a geometric distribution is assumed for the return process. However, given the memoryless property of such distribution, and the nature of any dynamic programming model (where decisions are taken at every decision epoch, where the system state is updated), the amount of returns follow a binomial distribution, such that the expected amount of returns $E(x_{t+1})$ can be obtained as shown.

Also, the variability for such returns can be modeled as:

$$Var(x_{t+1}) = n_t r (1 - r) \quad (4)$$

Which implies that such variability increases as n_t increases, for fixed r . Then, the variability in the return volume increases as the number of outstanding units increases.

Also, if a greater value for r is defined, such variability also increases as r gets closer to 0.5 (above such value the variability value decreases in the same proportion). However, as Rogers and Tibben-Lembke (1999) show, the return rate in most industries is between zero and 0.3, approaching 0.5 only in some specific industry sectors.

Assumption 3:

It is assumed that the firm's RL capacity is continuous; i.e., it can be added or subtracted in any quantity. This implies that the policy followed by any firm when adjusting its RL capacity can consider any amount of "capacity units". This applies for

the capacity investment cost, as well as the corresponding disinvestment and fixed capacity cost.

However (as it will be explained in Section 3.1.3), the state space for the capacity is discrete, given that the capacity adjustment will equal the expected returns in the next period defined in equation (2).

Then, considering the functions defined for the expected amount of returns and its variability, each of the scenarios proposed in section 2 can be modeled by defining the values for L , r , M , as well as a particular sales function.

Using this notation and assumptions, the MDM is defined in the next section.

3.1. Model Definition.

3.1.1. Decision epochs.

$$t \in \{1, 2, \dots, T - 1\}$$

Where decision epoch t represents the end of period t . Time T corresponds to the end of the problem horizon, where no decision is taken. Also, T is defined as follows:

$$T = L + W \tag{5}$$

Where as mentioned before, L represents the length of the product life cycle, and W is the time length defined by the firm to continue managing the returns for the product analyzed, after the last sale was made. Such length can usually be defined in terms of the service level for such returns stated by the firm, which ensures a warranty or accomplishment of the legal requirements for managing returned products after period L , when the last sale is experienced.

3.1.2. States.

The system state in each decision epoch t is defined as:

$$(k_t, w_t) \text{ for } t = 1, 2, \dots, T$$

Where k_t represents the RL capacity owned by the firm during period t , measured in units per period. As mentioned before, w_t represents the number of units that have been historically returned through the RL channel at the end of period t ; i.e.:

$$w_t = \sum_{i=1}^t x_i \quad (6)$$

The system states are partially ordered according to w_t (see Section 3.3.2).

3.1.3. Actions.

Given that the purpose of the MDM is to determine whether and when to outsource, it is assumed that at the end of any period t , either of the next two actions can be taken:

$a=0$ Continue performing the RL activities internally by updating the firm's capacity to the expected amount of returns in the next period:

$$k_{t+1} = E(x_{t+1}) = n_t r \quad (7)$$

Given that n_t is an integer and that the capacity levels k_{t+1} are adjusted according to equation (7), then the problem has a discrete state space for $a = 0$.

$a=1$ Follow an outsourcing strategy for the RL activities performed by the firm, by having a 3PRLP performing such activities and taking the firm's RL capacity to zero; i.e., $k_{t+1}=0$.

Given that RL does not represent a core activity for the firm, it is also assumed that once the outsourcing decision is taken, it remains in place for the rest of the problem horizon.

3.1.4. Transition Probabilities.

As it was already shown before, the returns follow a binomial distribution, and given that the sales function is also known, the transition probability values between states are defined as:

$$p_{t+1}((k_{t+1}, w_{t+1})|(k_t, w_t), a) \quad \text{for } a = \{0, 1\} \quad (8)$$

Where for $a=0$ we have:

$$p_{t+1}((n_t r, w_t + j)|(k_t, w_t), 0) = \begin{cases} \binom{n_t}{j} r^j (1-r)^{n_t-j} & \text{for } j = 0, 1, \dots, n_t \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

from the binomial distribution defined for the return process.

For $a=1$ we have:

$$p_{t+1}((0, w_t + j)|(k_t, w_t), 1) = \begin{cases} \binom{n_t}{j} r^j (1-r)^{n_t-j} & \text{for } j = 0, 1, \dots, n_t \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

from the binomial distribution defined for the return process.

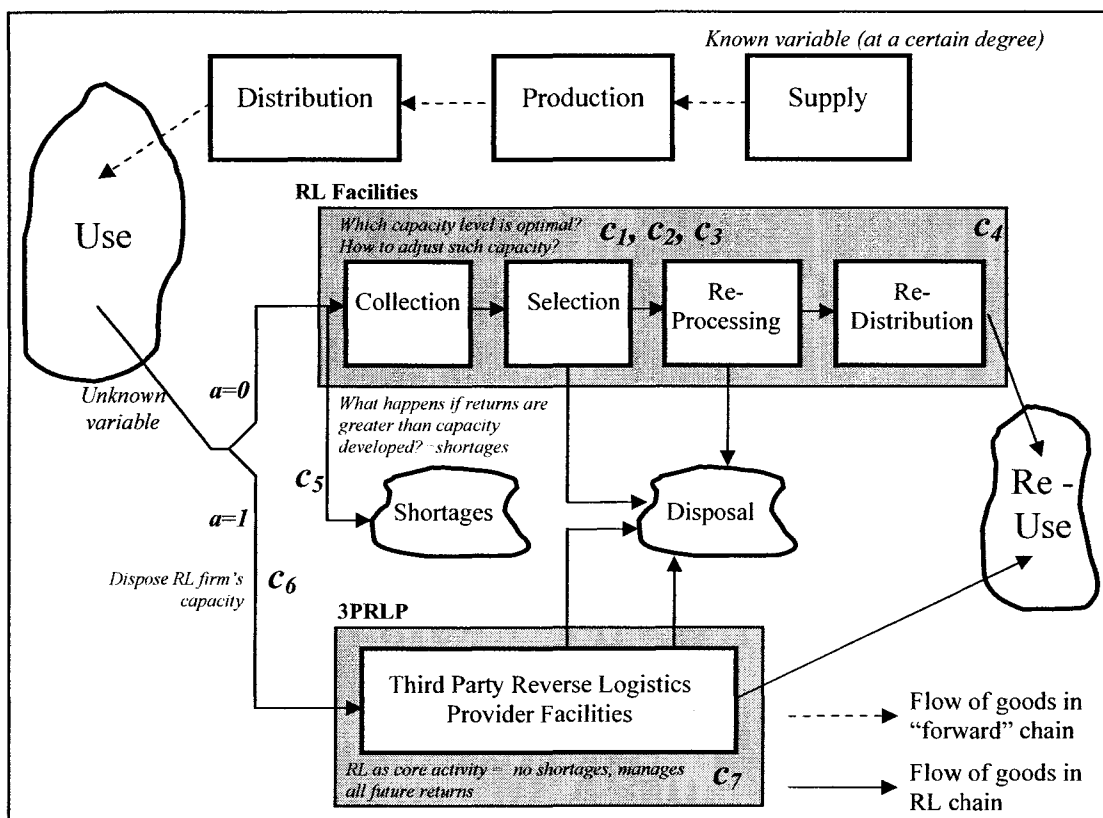
3.1.5. Rewards.

Let us define the following set of costs, where a capacity unit represents firm's ability to process one returned item:

- c_1 : Unit investment cost for increasing the firm's capacity (\$/capacity unit).
- c_2 : Unit disinvestment cost (\$/capacity unit).
- c_3 : Fixed internal cost (\$/capacity unit/period).
- c_4 : Unit internal labor cost (\$/unit).
- c_5 : Unit shortage cost (\$/unit).
- c_6 : Unit salvage value (\$/capacity unit).
- c_7 : Unit outsourcing cost (\$/unit).

Where $c_1, c_3, c_4, c_5, c_7 > 0$ because they represent costs for the firm, while c_2, c_6 are unrestricted in sign, which implies that neither the contracting capacity cost nor the salvage value are strictly positive or negative; i.e., there is no need to assume they will be an income or not. Figure 5 shows where these costs are located in the RL chain.

Figure 5. Relationship between RL chain and costs considered in MDM.



Given that (as mentioned before) RL does not represent a core activity for the firm, profits from remanufacturing are not considered. The next relationships are assumed between these cost parameters:

$$c_1 \geq |c_2| \quad (11)$$

$$c_3 \geq c_2 \quad (12)$$

$$c_4 < c_7 \quad (13)$$

$$c_7 < c_5 \quad (14)$$

$$c_5 \geq c_1 + c_3 + c_4 \quad (15)$$

First, (11) implies that what is invested/obtained when capacity is contracted, is less than what was invested to expand it; i.e., there can be no profit from expanding and contracting capacity. Equation (12) implies that the cost of decreasing the firm's capacity is less than the cost of maintaining it for an additional period.

Also, (13) because c_7 has to cover both fixed and variable costs for the 3PRLP, where c_4 consists only of the variable cost for the firm. But if economies of scale are considered (as should be, given that RL is a core activity for the 3PRLP) fixed costs per unit for the 3PRLP are less than fixed costs per unit for the firm. Also, (14) because otherwise, all the 3PRLP's potential clients could keep their own capacity low and just pay the shortage cost rather than following an outsourcing option.

On the other hand, equation (15) represents a motivation to develop internal capacity, given that the total internal cost of having the capacity for one additional period and then processing one additional unit is less than the shortage cost for such unit.

With these cost parameters, the following cost structure is defined for actions $a=\{0,1\}$.

For $a=0$, we have:

-Investment cost:

$$C_1(k_t, w_t) = c_1(n_t r - k_t)^+$$

-Disinvestment cost:

$$C_2(k_t, w_t) = c_2(k_t - n_t r)^+$$

-Fixed internal cost:

$$C_3(k_t, w_t) = c_3(n_t r)$$

-Expected Internal labor cost:

$$C_4(k_t, w_t) = c_4(E[\min(X, n_t r)]) \quad \text{where } X \text{ Bin}(n_t, r)$$

-Expected Shortage cost:

$$C_5(k_t, w_t) = c_5(E[(X - n_t r)^+]) \quad \text{where } X \text{ Bin}(n_t, r)$$

Where it is assumed that any unit that was not managed through the RL system in the period it was taken back, is lost and will not be remanufactured later. Then, this shortage cost reflects precisely the economic impact for such a situation.

Then, the total expected reward for $a=0$ is defined as follows:

$$\begin{aligned} R_{t+1}((k_t, w_t), 0) = & -c_1[n_t r - k_t]^+ - c_2[k_t - n_t r]^+ - c_3[n_t r] - \\ & c_4 \left[\sum_{j=0}^{n_t} (\min(j, n_t r)) \cdot p_{t+1}((n_t r, w_{t+1} = w_t + j)(k_t, w_t), 0) \right] - \\ & c_5 \left[\sum_{j=0}^{n_t} (\max(j - n_t r, 0)) \cdot p_{t+1}((n_t r, w_{t+1} = w_t + j)(k_t, w_t), 0) \right] \end{aligned}$$

For $a=1$, we have the next costs:

-Salvage value:

$$C_6(k_t, w_t) = c_6(k_t)$$

Which implies that, when the outsourcing decision is taken, the firm's RL capacity is taken to zero, situation that will last for the rest of the planning horizon.

-Expected Outsourcing cost:

$$C_7(k_t, w_t) = c_7\left(n_t + \sum_{l=t+1}^T s_l\right), \quad \text{where } \sum_{l=t+1}^T s_l = 0 \text{ if } t+1 > L$$

This cost corresponds to the payment made to the 3PRLP for the expected returns from period $t+1$ onwards. Recall that, given that RL is not a core activity, it is assumed that when taken, the outsourcing option will remain for the rest of the planning horizon. Recall also from assumption 1, that the future sales can also be estimated, according to the sales function and M value defined.

This function also implies that the 3PRLP has infinite capacity, given the fact that RL does represent a core activity for it.

Then, the total expected reward for $a=1$ is defined as follows:

$$R_{t+1}((k_t, w_t), 1) = c_6(k_t) - c_7\left(n_t + \sum_{l=t+1}^T s_l\right), \quad \text{where } \sum_{l=t+1}^T s_l = 0 \text{ if } t+1 > L$$

Also, we have the terminal reward in period T :

$$R_T(k_T, w_T, a) = c_6(k_T) - c_5(n_T), \text{ for } a = \{0,1\} \text{ and } k_T > 0$$

Which implies that the RL capacity defined by the firm is taken to zero in the last period, incurring the corresponding salvage value. Also, this function reflects the cost incurred by not being able to remanufacture any expected returned unit during period T or later.

3.2. System dynamics.

Recalling the MDM defined above, we can identify that during each period t the system:

1. Has a facility of size k_{t-1} at the beginning of such period, w_{t-1} units have historically been returned, and there are n_{t-1} units that are still in the market (were already sold and have not been returned);
2. Computes the expectation $E(x_t)=n_{t-1}r$ for the returns, as well as the corresponding expected internal reward $R_t((k_{t-1}, w_{t-1}), 0)$, and expected outsourcing reward $R_t((k_{t-1}, w_{t-1}), 1)$;
3. Applies a control $\delta_t(k_{t-1}, w_{t-1})=\{0, 1\}$;
4. If $\delta_t(k_{t-1}, w_{t-1})=0$, k_t is set equal to $E(x_t)$ and the firm incurs either an investment cost $C_1(k_{t-1}, w_{t-1})$, or a disinvestment cost $C_2(k_{t-1}, w_{t-1})$ by adjusting such capacity, as well as a fixed cost $C_3(k_{t-1}, w_{t-1})$;
5. If $\delta_t(k_{t-1}, w_{t-1})=1$, k_t is set equal to zero and the firm incurs a salvage value $C_6(k_{t-1}, w_{t-1})$;
6. Experiences a random amount of returns x_t , which determines the new system state $(k_t, w_t = w_{t-1} + x_t)$, as well as an amount of sales s_t , which determines the new cumulative sales level for the firm ($S_t=S_{t-1}+s_t$).
7. Incurs an internal/shortage cost $C_4(k_t, w_t)/C_5(k_t, w_t)$ or outsourcing cost $C_7(k_t, w_t)$.

Given an initial system state $(k_0, w_0=0)$, the problem is to find a sequence of decision functions $\{\delta_1^*(k_0, w_0), \delta_2^*(k_1, w_1), \dots, \delta_T^*(k_{T-1}, w_{T-1})\}$ that maximizes the total RL reward.

The optimal policy is obtained by solving recursively:

$$u_t(k_t, w_t) = \max \left\{ \begin{array}{l} -C_1(k_t, w_t) - C_2(k_t, w_t) - C_3(k_t, w_t) - C_4(k_t, w_t) - C_5(k_t, w_t) + \\ \sum_{j=0}^{n_t} p_{t+1}((n_t r, w_t + j))(k_t, w_t, 0) u_{t+1}(n_t r, w_t + j), \\ C_6(k_t, w_t) - C_7(k_t, w_t) \end{array} \right\}$$

where $u_t(k_t, w_t)$ represents the maximum expected reward from being in state (k_t, w_t) onwards. This reward is obtained when taking action $a_t^*(k_t, w_t)$, which represents the optimal action a to take when being in state (k_t, w_t) .

3.3. Characteristics of the optimal policy to be found.

As it was already mentioned before, RL does not represent a core activity for the firm, given that the main purpose of any company is not to manage the flow of the products taken back from the sale point, but to deliver such products to its customers.

This implies that the outsourcing option for RL is mostly identified as a “take it or leave it” alternative, given that the firm will not be continually changing its strategy to manage such returns. The first idea would be to solve the MDM proposed by using backward induction, which will allow us to identify the convenience of the outsourcing option in each decision epoch. However, as it was explained in the qualitative analysis performed in section 2, no firm will be interested in changing back and forth between an internal and outsourcing strategy during the analysis horizon. Instead, and given the nature of the RL functions, it will be interested in identifying whether or not to follow an outsourcing strategy during such cycle.

Then, it can be said in terms of the MDM proposed, that a monotone deterministic optimal policy should be identified; i.e., either to outsource or not such activities during the horizon analysis. The value of identifying an optimal monotone deterministic policy can be clearly explained by considering the definition stated by Puterman (1994) for this type of policies. This author mentions that these policies imply that, when the system defined considers only two actions to be taken (outsourcing or not the RL functions in our case) the problem can be reduced to identifying the threshold above which it is optimal to take one of such actions. If this threshold is not crossed, then it is optimal to continue taking the same action that was taken in the previous period.

As Puterman (1994) also states, such a threshold represents nothing but a control limit policy. Such deterministic Markov policy is composed of decision rules of the form:

$$d_i(s) = \begin{cases} a_1 & s < s^* \\ a_2 & s \geq s^* \end{cases}$$

Where a_1 and a_2 are distinct actions, s is the system state and s^* is the threshold or control limit. If we establish that such policies are optimal for the MDM proposed, the problem of finding an optimal policy reduces to that of determining $(k_b, w_t)^*$ in our model; i.e., the threshold above which outsourcing (action $a=1$) the RL functions is optimal for the scenario considered.

This threshold simplifies the solution procedure, because when it is known that an optimal policy of a specific form exists, specialized algorithms can be developed to search only among policies that have that form. This avoids the need for less efficient general algorithms like backward induction.

Then, the problem can be reduced to identifying the set of conditions under the ones a monotone deterministic nondecreasing policy is optimal; i.e., if such conditions are satisfied, then there is a threshold above which it is optimal to follow an outsourcing strategy ($a=1$). Below such threshold, the firm should continue performing the RL activities internally ($a=0$).

3.3.1. Hypothesis 3 rewritten.

Considering this information and the model proposed, hypothesis 3 stated before can be rewritten as follows:

Hypothesis 3 rewritten:

For products with short life cycle and high return variability, the threshold above which outsourcing is optimal is more likely to be crossed, while for products with long life cycle and low return variability, this threshold-crossing is less likely.

Next, the conditions for the existence of an optimal deterministic nondecreasing policy will be defined in terms of the MDM proposed.

3.3.2. Conditions for identifying a monotone deterministic nondecreasing policy as optimal.

As it is stated by Puterman (1994), there exists a set of conditions that ensure that optimal policies are monotone in the system state. For such a concept to be meaningful, it is required that the state have a physical interpretation and some natural ordering. The expression “monotone policy” refers to a monotone deterministic Markov policy.

Recall from section 3.1.2., that for the MDM proposed the states are partially ordered in terms of the cumulative returned units w_t ; i.e., the higher the amount of units historically returned, the higher the system’s state. This ordering criterion represents just a partial ordering for the system state because there may exist two or more system states with the same cumulative returned units w_t , but with a different RL system capacity. This ordering can be algebraically stated as follows:

For each t , let the states (k_t, w_t) be strictly partially ordered according to the next criteria:

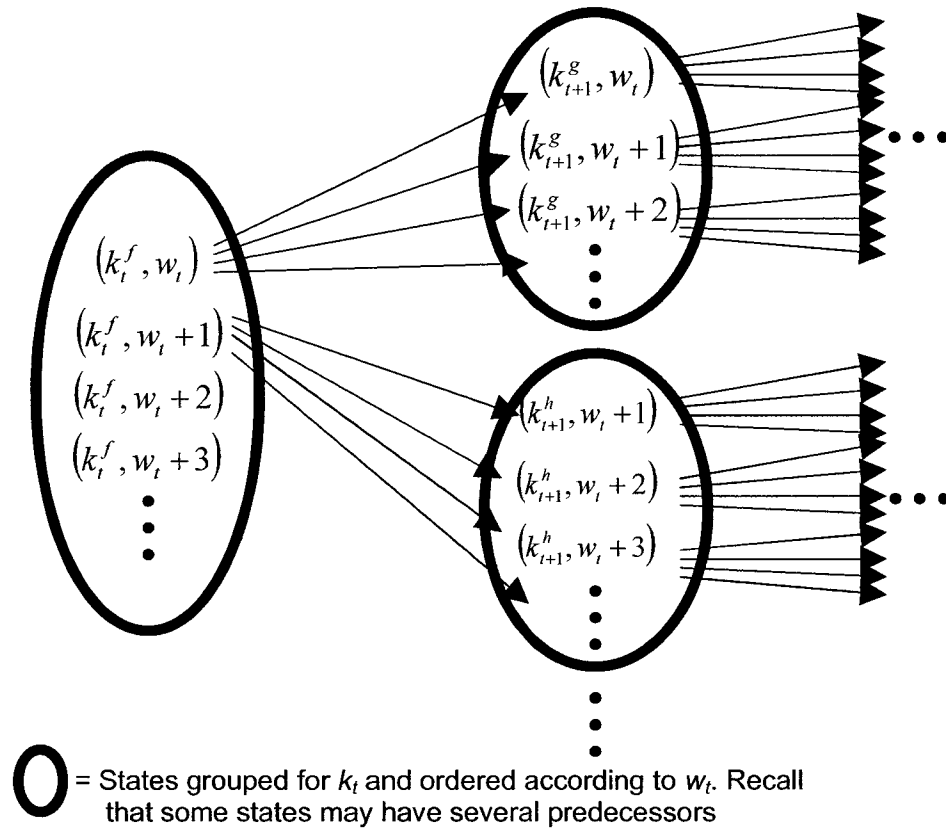
1. For every t , group the states where k_t has a particular value (defined as $k_t^f, k_t^g, k_t^h \dots$).
2. For each group generated, generate a logical ordering for the states according to w_t , i.e; the larger w_t is, the greater the state is.

This strict partial ordering implies that:

$$(k^1, w^1) \prec (k^2, w^2) \Leftrightarrow k^1 = k^2 \text{ and } w^1 < w^2$$

which can also be graphically seen on Figure (6).

Figure 6. Criterion followed for a strict partial state ordering.



Additional to the partial ordering defined, the next cumulative probability has also to be defined in order to identify the conditions for a monotone nondecreasing policy:

$$q_i((k_i, w_i)|(k_{i-1}, w_{i-1}), a) = \sum_{w_i = w_{i-1}}^{n_{i-1}} p_i((k_i, w_i)|(k_{i-1}, w_{i-1}), a)$$

Where for $a=0$ we have:

$$q_i((n_{i-1}r, w_i)|(k_{i-1}, w_{i-1}), 0) = \begin{cases} \sum_{j=w_i-w_{i-1}}^{n_{i-1}} \binom{n_{i-1}}{j} r^j (1-r)^{n_{i-1}-j} & \text{for } w_i \geq w_{i-1} \\ 1 & \text{for } w_i < w_{i-1} \end{cases}$$

For $a=1$ we have:

$$q_t((0, w_t)|(k_{t-1}, w_{t-1}), 1) = \begin{cases} \sum_{j=w_t-w_{t-1}}^{n_{t-1}} \binom{n_{t-1}}{j} r^j (1-r)^{n_{t-1}-j} & \text{for } w_t \geq w_{t-1} \\ 1 & \text{for } w_t < w_{t-1} \end{cases}$$

Finally, the concepts of superadditive and subadditive functions have also to be defined. To do so, let X and Y be partially ordered sets and $g(x, y)$ a real-valued function on $X \times Y$. It is said that g is superadditive if for $x^- \leq x^+$ in X and $y^- \leq y^+$ in Y :

$$g(x^+, y^+) + g(x^-, y^-) \geq g(x^+, y^-) + g(x^-, y^+)$$

If the reverse inequality above holds, $g(x, y)$ is said to be subadditive.

Considering this information, the conditions stated in a Theorem shown by Puterman (1994) for the existence of a nondecreasing monotone policy are:

1. $R_t((k, w), a)$ is nondecreasing in (k, w) for $a = \{0, 1\}$,
2. $q_t((k_t, w_t = w_t) | (k, w), a)$ is nondecreasing in (k, w) for all w_t and $a = \{0, 1\}$,
3. $R_t((k, w), a)$ is a superadditive function on $(k, w) \times a$,
4. $q_t(q_t((k_t, w_t = w_t) | (k, w), a))$ is a superadditive function on $(k, w) \times a$, and
5. $R_T(k_T, w_T)$ is nondecreasing in (k, w) .

When all of these conditions are satisfied, there exists a control limit policy that is optimal; i.e., given that only two actions can be taken, for each set of partially ordered states there exists a particular state $(k_b, w_t)^*$ with the property that, if (k, w) exceeds $(k_b, w_t)^*$, i.e., if the amount of cumulative returned units goes above w_t^* , then the optimal decision is to outsource the RL functions to the 3PRLP for the rest of the planning horizon; and if $(k, w) \leq (k_b, w_t)^*$, then it is optimal to continue performing the RL functions internally for another period.

3.4. Requirements in the MDM for the existence of a Monotone Nondecreasing Policy

In order to prove these five conditions for the MDM developed, the next lemma will be used:

Lemma 1.

Suppose X_n is binomial with parameters n and r , where $n = 1, 2, \dots$ and $0 < r < 1$. Let $\mu_n = E(X_n) = nr$. Then for any n and $l = 1, 2, \dots, n-1$,

$$(a) \quad E[(X_n - \mu_n)^+] \geq E[(X_l - \mu_l)^+]$$

$$(b) \quad E[\min(X_n, \mu_n)] \geq E[\min(X_l, \mu_l)]$$

$$(c) \quad \text{For any integer } m \text{ such that } 0 \leq m \leq l, P\{X_n > m\} \geq P\{X_l > m\}$$

Proof:

(a) Suppose $l = n-1$ and consider

$$E[(X_n - nr)^+ - (X_{n-1} - (nr - r))^+] = E[E[(X_n - nr)^+ - (X_{n-1} - (nr - r))^+ | X_{n-1}]]$$

where $X_n = X_{n-1} + U$

where $U = 1$ with probability r and 0 otherwise. There are three possible cases for $X_{n-1} = m$:

Table 5. Cases considered for Case (a) in Lemma 1.

Case	Conditioning argument
1	$m < nr - r$
2	$nr - r \leq m < nr$
3	$nr \leq m$

In the first case, $E[(X_n - nr)^+ - (X_{n-1} - (nr - r))^+ | X_{n-1} = m]$ can be reduced as follows:

$$\begin{aligned}
& E\left[(X_n - nr)^+ - (X_{n-1} - (nr - r))^+ \mid X_{n-1} = m\right] \\
&= E\left[(m + U - nr)^+\right] - 0 \\
&= r(m + 1 - nr)^+ + (1 - r)(m - nr)^+ \\
&= r(m + 1 - nr)^+
\end{aligned}$$

given that $nr > m$. Then, this case yields a nonnegative result. In the second case, the expression can be reduced as follows:

$$\begin{aligned}
& E\left[(X_n - nr)^+ - (X_{n-1} - (nr - r))^+ \mid X_{n-1} = m\right] \\
&= E\left[(m + U_n - nr)^+\right] - (m - nr + r) \\
&= r(m + 1 - nr)^+ + (1 - r)(m - nr)^+ - m + nr - r \\
&= r(m + 1 - nr) - m + nr - r \\
&= (nr - m)(1 - r)
\end{aligned}$$

where, given that $nr > m$, such expression is also nonnegative. Finally, for the third case, the expression can be reduced to:

$$\begin{aligned}
& E\left[(X_n - nr)^+\right] - E\left[(X_1 - (nr - r))^+\right] \\
&= E\left[(m + U_n - nr)^+\right] - (m - nr + r) \\
&= r(m + 1 - nr)^+ + (1 - r)(m - nr)^+ - m + nr - r \\
&= r(m + 1 - nr) + (1 - r)(m - nr)^+ - m + nr - r \\
&= 0
\end{aligned}$$

given that $m \geq nr$. Then, in order to complete the conditioning argument:

$$\begin{aligned}
& E\left[(X_n - nr)^+\right] - E\left[(X_{n-1} - (nr - r))^+\right] \\
&= E\left[E\left[(X_n - nr)^+ - (X_{n-1} - (nr - r))^+ \mid X_{n-1}\right]\right] \\
&= \sum_{m=0}^{n-1} E\left[(X_n - nr)^+ - (X_{n-1} - (nr - r))^+ \mid X_{n-1} = m\right] P(X_{n-1} = m) \geq 0
\end{aligned}$$

which yields a nonnegative result, as a consequence of multiplying a set of nonnegative values by their corresponding probabilities.

Then, considering that:

$$E[(X_n - \mu_n)^+] - E[(X_{n-1} - \mu_{n-1})^+] \geq 0$$

and since n is arbitrary:

$$\begin{aligned} E[(X_{n-1} - \mu_{n-1})^+] - E[(X_{n-2} - \mu_{n-2})^+] &\geq 0 \\ E[(X_{n-2} - \mu_{n-2})^+] - E[(X_{n-3} - \mu_{n-3})^+] &\geq 0 \\ \dots \end{aligned}$$

We have, by the transitive property:

$$E[(X_n - \mu_n)^+] - E[(X_l - \mu_l)^+] \geq 0 \text{ for any } l = 1, 2, \dots, n-1$$

which completes the proof.

(b) Suppose $l = n-1$ and consider

$$\begin{aligned} E[\min(X_n, nr) - \min(X_{n-1}, (nr-r))] &= E[E[\min(X_n, nr) - \min(X_{n-1}, (nr-r)) | X_{n-1}]] \\ \text{where } X_n &= X_{n-1} + U \end{aligned}$$

where $U = 1$ with probability r and 0 otherwise. Suppose $X_{n-1} = m < nr - r$. Then

$E[\min(X_n, nr) - \min(X_{n-1}, (nr-r)) | X_{n-1} = m]$ can be reduced as follows:

$$\begin{aligned} &E[\min(X_n, nr) - \min(X_{n-1}, (nr-r)) | X_{n-1} = m] \\ &= E[\min(m+U, nr)] - m \\ &= r \min(m+1, nr) + (1-r) \min(m, nr) - m \\ &= r \min(m+1, nr) + (1-r)m - m \end{aligned}$$

Given that $m < nr - ir \Rightarrow m < nr$. Also, $nr > m + r \Rightarrow \min(m + 1, nr) \geq m + r$, so $r \min(m + 1, nr) + (1 - r)m - m \geq r(m + r) + (1 - r)m - m = r^2$.

On the other hand, suppose $X_{n-1} = m \geq nr - r$.

Then $E[\min(X_n, nr) - \min(X_{n-1}, (nr - r)) | X_{n-1} = m]$ can be reduced as follows:

$$\begin{aligned} & E[\min(X_n, nr) - \min(X_{n-1}, (nr - r)) | X_{n-1} = m] \\ &= E[\min(m + U_n, nr)] - nr + r \\ &= r \min(m + 1, nr) + (1 - r) \min(m, nr) - nr + r \\ &= rnr + (1 - r) \min(m, nr) - nr + r \end{aligned}$$

Because $m \geq nr - r \Rightarrow m + 1 \geq nr$. Also, $\min(m, nr) \geq nr - r$, which implies that $rnr + (1 - r) \min(m, nr) - nr + r \geq nr^2 + (1 - r)(nr - r) - nr + r = r^2$.

In order to complete the conditioning argument:

$$\begin{aligned} & E[\min(X_n, nr) - \min(X_l, nr - r)] \\ &= E[E[\min(X_n, nr) - \min(X_l, nr - r) | X_l]] \\ &= \sum_{m=0}^{n-1} E[\min(X_n, nr) - \min(X_l, nr - r) | X_l = m] P(X_l = m) \end{aligned}$$

which implies that such expression yields a nonnegative result. Then:

$$E[\min(X_n, \mu_n)] - E[\min(X_{n-1}, \mu_{n-1})] \geq 0.$$

and since n is arbitrary:

$$\begin{aligned} & E[\min(X_{n-1}, \mu_{n-1})] - E[\min(X_{n-2}, \mu_{n-2})] \geq 0 \\ & E[\min(X_{n-2}, \mu_{n-2})] - E[\min(X_{n-3}, \mu_{n-3})] \geq 0 \\ & \dots \end{aligned}$$

We have, by the transitive property:

$$E[\min(X_n, \mu_n)] - E[\min(X_l, \mu_l)] \geq 0 \text{ for any } l = 1, 2, \dots, n-1$$

which completes the proof.

(c) Suppose $l = n-1$ and consider

$$P\{X_n > m\} - P\{X_{n-1} > m\}$$

where m is an integer. Then:

$$\begin{aligned} P\{X_n > m\} - P\{X_{n-1} > m\} &= [P\{X_{n-1} > m\} + P\{X_{n-1} = m\}P\{U = 1\}] - P\{X_{n-1} > m\} \\ &= \binom{n-1}{m} r^{m+1} (1-r)^{n-1-m} > 0, \quad \text{where } X_n = X_{n-1} + U \end{aligned}$$

which implies that the probability of experiencing more than m successes is greater when one additional trial is added to the sequence of Bernoulli trials:

$$P\{X_n > m\} - P\{X_{n-1} > m\} > 0$$

and since n is arbitrary:

$$\begin{aligned} P\{X_{n-1} > k\} - P\{X_{n-2} > k\} &> 0 \\ P\{X_{n-2} > k\} - P\{X_{n-3} > k\} &> 0 \\ &\dots \end{aligned}$$

We have, by the transitive property:

$$P\{X_n > k\} - P\{X_l > k\} > 0 \text{ for any } l = 1, 2, \dots, n-1$$

which completes the proof.

Next, the requirements to satisfy the five conditions stated by Puterman (1994) for the existence of a monotone nondecreasing policy, will be described in terms of the Markov Decision Model (MDM) developed.

3.4.1. Condition 1.

This condition implies that the cost of either decision increases with the number of items sold but not yet returned. For $a=1$, this condition is stated as follows:

$$\begin{aligned} R_t((k_{t-1}, w_{t-1}), 1) &\leq R_t((k_{t-1}, w_{t-1} + i), 1), & 1 \leq i \leq n_{t-1} \\ c_6 k_{t-1} - c_7 \left(n_{t-1} + \sum_{l=t}^T s_l \right) &\leq c_6 k_{t-1} - c_7 \left(n_{t-1} - i + \sum_{l=t}^T s_l \right) \\ n_{t-1} &\geq n_{t-1} - i, & 1 \leq i \leq n_{t-1} \end{aligned}$$

where c_6 *urs* and $c_7 > 0$. This implies that such condition is always satisfied for $a=1$.

For the case of $a=0$ (to continue performing the RL functions internally for another period), this condition is stated as follows:

$$\begin{aligned} R_{t+1}((k_t, w_t), 0) &\leq R_{t+1}((k_t, w_t + i), 0) \quad 1 \leq i \leq n_t \\ -c_1(n_t r - k_t)^+ - c_2(k_t - n_t r)^+ - c_3 n_t r - c_4 &\left[\sum_{j=0}^{n_t} (\min(j, n_t r)) \bullet p_{t+1}(((n_t r, w_{t+1} = w_t + j)(k_t, w_t), 0)) \right] - \\ c_5 &\left[\sum_{j=0}^{n_t} (\max(j - n_t r, 0)) \bullet p_{t+1}(((n_t r, w_{t+1} = w_t + j)(k_t, w_t), 0)) \right] \leq \\ -c_1((n_t - i)r - k_t)^+ - c_2(k_t - (n_t - i)r)^+ - c_3(n_t - i)r - \\ c_4 &\left[\sum_{j=0}^{n_t - i} (\min(j, (n_t - i)r)) \bullet p_{t+1}(((n_t - i)r, w_{t+1} = w_t + i + j)(k_t, w_t + i), 0)) \right] - \\ c_5 &\left[\sum_{j=0}^{n_t - i} (\max(j - (n_t - i)r, 0)) \bullet p_{t+1}(((n_t - i)r, w_{t+1} = w_t + i + j)(k_t, w_t + i), 0)) \right] \quad 1 \leq i \leq n_t \end{aligned}$$

Where c_2 *urs* and $c_1, c_3, c_4, c_5 > 0$. This previous inequality implies that for a fixed capacity k_t , the expected internal RL reward (the RL cost) will be greater when the

cumulative amount of returned units w_t is greater. Such inequality can also be rewritten as:

$$\begin{aligned}
& -c_1 \left((n_t r - k_t)^+ - ((n_t - i)r - k_t)^+ \right) - c_2 \left((k_t - n_t r)^+ - (k_t - (n_t - i)r)^+ \right) - c_3 r i - \\
& c_4 \left[\sum_{j=0}^{n_t} (\min(j, n_t r)) \cdot p_{t+1} \left((n_t r, w_{t+1} = w_t + j)(k_t, w_t, 0) \right) - \right. \\
& \left. \sum_{j=0}^{n_t - i} (\min(j, (n_t - i)r)) \cdot p_{t+1} \left(((n_t - i)r, w_{t+1} = w_t + i + j)(k_t, w_t + i, 0) \right) \right] - \\
& c_5 \left[\sum_{j=0}^{n_t} (\max(j - n_t r, 0)) \cdot p_{t+1} \left((n_t r, w_{t+1} = w_t + j)(k_t, w_t, 0) \right) - \right. \\
& \left. \sum_{j=0}^{n_t - i} (\max(j - (n_t - i)r, 0)) \cdot p_{t+1} \left(((n_t - i)r, w_{t+1} = w_t + i + j)(k_t, w_t + i, 0) \right) \right] \leq 0, \quad 1 \leq i \leq n_t,
\end{aligned}$$

This expression can be reduced as follows:

$$\begin{aligned}
& -c_1 \left((n_t r - k_t)^+ - ((n_t - i)r - k_t)^+ \right) - c_2 \left((k_t - n_t r)^+ - (k_t - (n_t - i)r)^+ \right) - c_3 r i - \\
& c_4 [E[\min(X, n_t r)] - E[\min(Y, (n_t - i)r)]] - c_5 [E[(X - n_t r)^+] - E[(Y - (n_t - i)r)^+]] \leq 0, \quad 1 \leq i \leq n_t,
\end{aligned}$$

Where, based on parts (a) and (b) of Lemma 1, it follows that the elements that multiply c_4 and c_5 are nonnegative.

In order to analyze the previous inequality, the next possible cases need to be considered:

- 1) $n_t r < k_t$
- 2) $k_t \leq (n_t - i)r \quad i = 1, 2, \dots, n_t - 1$
- 3) $(n_t - i)r \leq k_t \leq n_t r$

In the first case, the inequality can be reduced to the next expression:

$$\begin{aligned}
& -ir(c_3 - c_2) - c_4 [E[\min(X, n_t r)] - E[\min(Y, (n_t - i)r)]] \\
& - c_5 [E[(X - n_t r)^+] - E[(Y - (n_t - i)r)^+]] \leq 0, \quad 1 \leq i \leq n_t,
\end{aligned}$$

which implies that this condition is satisfied, given equation (12).

In the second case, the inequality can be reduced to:

$$\begin{aligned} & -ir(c_1 + c_3) - c_4 [E[\min(X, n_t r)] - E[\min(Y, (n_t - i)r)]] \\ & - c_5 [E[(X - n_t r)^+] - E[(Y - (n_t - i)r)^+]] \leq 0, 1 \leq i \leq n_t \end{aligned}$$

which implies that this inequality is already satisfied.

Finally, this inequality can be written as follows for the last case:

$$\begin{aligned} & -ir(c_3 - c_2) + (k_t - n_t r)(c_1 + c_2) - c_4 [E[\min(X, n_t r)] - E[\min(Y, (n_t - i)r)]] \\ & - c_5 [E[(X - n_t r)^+] - E[(Y - (n_t - i)r)^+]] \leq 0, 1 \leq i \leq n_t \end{aligned}$$

which also implies that it is satisfied, given equations (11) and (12).

3.4.2. Condition 2.

This condition implies that it is more likely to meet or exceed a given number of cumulative returns in the next period, if a higher number of returns have been experienced up to the current period. Then, this condition requires that:

$$q_t((k_t, w_t = w_l)(k_{t-1}, w_{t-1}), a) \leq q_t((k_t, w_t = w_l)(k_{t-1}, w_{t-1} + i), a) \quad 1 \leq i \leq n_{t-1}$$

which can be analyzed under the next three cases:

- 1) $w_t \leq w_{t-1}$
- 2) $w_{t-1} < w_t \leq w_{t-1} + i \quad 1 \leq i \leq n_{t-1}$
- 3) $w_{t-1} + i < w_t \quad 1 \leq i \leq n_{t-1}$

The first case implies the cumulative returns in $t-1$ are in both sides of the inequality greater or equal than w_t . Then, the probability that such cumulative returns will be equal or greater to w_t in the next period is 1; i.e., this condition is always satisfied as an equality in this case.

In the second case, the cumulative returns are already greater than w_t in the right hand side of the inequality ($w_t \leq w_{t-1} + i$). This implies that such probability equals 1, and in consequence, that such condition will be satisfied in this case regardless the value that w_{t-1} has; i.e., regardless the probability value in the left hand side of the inequality.

Finally, for the third case, this condition can be rewritten as follows:

$$\sum_{j=w_t-w_{t-1}}^{n_{t-1}} \binom{n_{t-1}}{j} r^j (1-r)^{n_{t-1}-j} \leq \sum_{j=w_t-w_{t-1}-i}^{n_{t-1}-i} \binom{n_{t-1}-i}{j} r^j (1-r)^{n_{t-1}-i-j}, \quad w_{t-1} < w_{t-1} + i < w_t, \quad 1 \leq i \leq n_{t-1}$$

which can be stated as:

$$1 - \sum_{j=0}^{w_t-w_{t-1}-1} \binom{n_{t-1}}{j} r^j (1-r)^{n_{t-1}-j} \leq 1 - \sum_{j=0}^{(w_t-w_{t-1}-1)-i} \binom{n_{t-1}-i}{j} r^j (1-r)^{n_{t-1}-i-j},$$

and reduced to the next expression:

$$\sum_{j=0}^{w_t-w_{t-1}-1} \binom{n_{t-1}}{j} r^j (1-r)^{n_{t-1}-j} \geq \sum_{j=0}^{w_t-w_{t-1}-1-i} \binom{n_{t-1}-i}{j} r^j (1-r)^{n_{t-1}-i-j}$$

which represents the comparison of the two random variables X and Y defined in Lemma 1, part (c). This condition is equivalent to say that:

$$P(X \leq w_t - w_{t-1} - 1) \geq P(Y \leq w_t - w_{t-1} - 1 - i)$$

Which is true if and only if the probability of having more than $w_t - w_t - w_{t-1}$ “failures” in n_{t-1} trials is greater or equal than the probability of having more than the same number of failures in $n_{t-1} - i$ trials, which was already proved in Lemma 1, part (c).

Then, it can be concluded that condition 2 is always satisfied for the MDM developed.

3.4.3. Condition 3.

$$R_{t+1}((k_t, w_t), 1) - R_{t+1}((k_t, w_t), 0) \leq R_{t+1}((k_t, w_t + i), 1) - R_{t+1}((k_t, w_t + i), 0)$$

This equation implies that for a fixed capacity k_t , the incremental effect on the RL reward of switching to an outsourcing strategy is greater when the cumulative amount of returned units w_t is greater. In other words, given a fixed capacity k_t , the difference between the internal and outsourcing RL cost is greater when the current cumulative returned amount of units is greater.

This Condition can be rewritten as:

$$c_1((n_t - i)r - k_t)^+ - (n_t r - k_t)^+ + c_2((k_t - (n_t - i)r)^+ - (k_t - n_t r)^+) + ic_7 - irc_3 \geq c_4[E[\min(X, n_t r)] - E[\min(Y, (n_t - i)r)]] + c_5[E[(X - n_t r)^+] - E[(Y - (n_t - i)r)^+]] \text{ for } 1 \leq i \leq n_t$$

Where, because of lemma A, parts (a) and (b), we already know that the expressions that multiply c_4 and c_5 are nonnegative.

In order to analyze the previous inequality, the next possible cases need to be considered:

$$\begin{array}{ll} \text{Case 1)} & n_t r < k_t \\ \text{Case 2)} & k_t \leq (n_t - i)r \\ \text{Case 3)} & (n_t - i)r \leq k_t \leq n_t r \end{array}$$

In the first case, the inequality is reduced to:

$$c_7 i - (c_3 - c_2)ir \geq c_4[E[\min(X, n_t r)] - E[\min(Y, (n_t - i)r)]] + c_5[E[(X - n_t r)^+] - E[(Y - (n_t - i)r)^+]] \quad (16)$$

Considering that $X = Y + \sum_{j=n_t-i+1}^{n_t} U_j$; i.e., the random variable Y corresponds to the first

$n_t - i$ trials with success probability r , while the random variable X corresponds to those

$n_t - i$ trials plus the remaining i to have n_t trials in total (all of them with the same success probability r), this inequality can be analyzed under the four possible cases shown in Table 6.

Table 6. Cases for Condition 3 when $n_t r < k_t$

Case	Value on the right-hand side of the inequality	Resulting inequality in the worst case:
1.1) $X > n_t r, Y > (n_t - i)r$	$c_4 i r + c_5 (X - Y - i r)$ Worst case: $X - Y = i$	$c_7 - c_5 \geq (c_3 - c_2)r - (c_5 - c_4)r$
1.2) $X > n_t r, Y \leq (n_t - i)r$	$n_t r (c_4 - c_5) + c_5 X - c_4 Y$ Worst case: $Y = n_t r - i r,$ $X = n_t r - i r + i$ Because $c_5 \geq c_4$	$c_7 - c_5 \geq (c_3 - c_2)r - (c_5 - c_4)r$
1.3) $X \leq n_t r, Y \leq (n_t - i)r$	$c_4 (X - Y)$ Worst case: $X - Y = i$	$c_7 - c_4 \geq (c_3 - c_2)r$
1.4) $X \leq n_t r, Y > (n_t - i)r$	$n_t r (c_5 - c_4) + c_4 X - c_5 Y + i r (c_4 - c_5)$ Worst case: $Y = (n_t - i)r,$ $X = n_t r$	$c_7 - c_4 r \geq (c_3 - c_2)r$, which is already included in $c_7 - c_4 \geq (c_3 - c_2)r$

This implies that, to satisfy Condition 3 when $n_t r < k_t$, the next two inequalities are required:

$$c_7 - c_5 \geq (c_3 - c_2)r - (c_5 - c_4)r \quad (17)$$

$$c_7 - c_4 \geq (c_3 - c_2)r \quad (18)$$

In the second case the inequality is reduced to:

$$c_7 i - (c_1 + c_3)ir \geq c_4 [E[\min(X, n_i r)] - E[\min(Y, (n_i - i)r)] + c_5 [E[(X - n_i r)^+] - E[(Y - (n_i - i)r)^+]]$$

Following the same analysis than before, it can be said that the resulting inequalities in the worst cases are the ones shown on Table 7.

Table 7. Cases for Condition 3 when $k_i \leq (n_i - i)r$

Case	Value on the right-hand side of the inequality	Resulting inequality in the worst case:
1.1) $X > n_i r, Y > (n_i - i)r$	$c_4 ir + c_5 (X - Y - ir)$ Worst case: $X - Y = i$	$c_7 - c_5 \geq (c_1 + c_3)r - (c_5 - c_4)r$
1.2) $X > n_i r, Y \leq (n_i - i)r$	$n_i r(c_4 - c_5) + c_5 X - c_4 Y$ Worst case: $Y = nr - ir,$ $X = nr - ir + i$ Because $c_4 < c_5$	$c_7 - c_5 \geq (c_1 + c_3)r - (c_5 - c_4)r$
1.3) $X \leq n_i r, Y \leq (n_i - i)r$	$c_4 (X - Y)$ Worst case: $X - Y = i$	$c_7 - c_4 \geq (c_1 + c_3)r$
1.4) $X \leq n_i r, Y > (n_i - i)r$	$n_i r(c_5 - c_4) + c_4 X - c_5 Y + ir(c_4 - c_5)$ Worst case: $Y = (n_i - i)r,$ $X = n_i r$	$c_7 - c_4 r \geq (c_1 + c_3)r$, which is already included in $c_7 - c_4 \geq (c_1 + c_3)r$

This implies that, to satisfy Condition 3 when $k_i < (n_i - i)r$, the next two inequalities are required:

$$c_7 - c_5 \geq (c_1 + c_3)r - (c_5 - c_4)r \quad (19)$$

$$c_7 - c_4 \geq (c_1 + c_3)r \quad (20)$$

Finally, in the third case the inequality is reduced to:

$$c_7 i - c_3 i r + c_1 (n_t r - k_t) + c_2 (k_t - (n_t - i)r) \geq c_4 [E[\min(X, n_t r)] - E[\min(Y, (n_t - i)r)]] + c_5 [E[(X - n_t r)^+] - E[(Y - (n_t - i)r)^+]], \text{ where } (n_t - i)r \leq k_t \leq n_t r$$

If this inequality is compared to inequality (16), it can be seen that it will be satisfied as long as:

$$c_2 (k_t - (n_t - i)r) + c_1 (n_t r - k_t) \geq c_2 i r, \quad \text{where } (n_t - i)r \leq k_t \leq n_t r$$

This expression can be reduced to:

$$(c_1 - c_2)(n_t r - k_t) \geq 0$$

which is always satisfied, given equation (11) and that $n_t r \geq k_t$ in this case.

Then, (17), (18), (19), and (20) are required to satisfy Condition 3. However, given (11), inequalities (19) and (20) are redundant. Also, given that $(c_5 - c_4) \geq (c_5 - c_4)r$, then (17) and (18) can be reduced to:

$$r \leq \frac{c_7 - c_5}{c_1 + c_3 + c_4 - c_5}$$

Which, given (14) and (15), can be also rewritten as:

$$r \leq \frac{c_5 - c_7}{c_5 - c_1 - c_3 - c_4} \quad (21)$$

which represents an upper limit on the return rate r , and the required inequality to satisfy Condition 3 (additional to the assumptions on the cost parameters stated on Section 3.1.5).

3.4.4. Condition 4.

This condition implies that the difference between the cumulative probability that returns are above a given number when taking the outsourcing option and when performing RL activities internally, is greater when the current returns are greater. This condition can be written as follows:

$$q_t((k_t, w_t)(k_{t-1}, w_{t-1}^+)1) - q_t((k_t, w_t)(k_{t-1}, w_{t-1}^+)0) \geq q_t((k_t, w_t)(k_{t-1}, w_{t-1}^-)1) - q_t((k_t, w_t)(k_{t-1}, w_{t-1}^-)0)$$

For $w_{t-1}^+ > w_{t-1}^-$. Given that such transition probability values do not depend on the current RL capacity (k_t), which is changed when the outsourcing decision is taken, this condition is satisfied as strict equality; i.e., given the next equalities satisfied for such transition probability values:

$$q_t((k_t, w_t)(k_{t-1}, w_{t-1}^+)1) = q_t((k_t, w_t)(k_{t-1}, w_{t-1}^+)0)$$

$$q_t((k_t, w_t)(k_{t-1}, w_{t-1}^-)1) = q_t((k_t, w_t)(k_{t-1}, w_{t-1}^-)0)$$

Then both sides of the inequality equal zero, which satisfies the requirement stated for this condition.

3.4.5. Condition 5.

This condition implies that the terminal reward is greater when the current amount of cumulative returns is greater. It can be written and reduced as follows:

$$R_T(k_T, w_T) \leq R_T(k_T, w_T + i), \quad 1 \leq i \leq n_T$$

$$c_6 k_T - c_5 n_T r \leq c_6 k_T - c_5 (n_T - i) r \quad 1 \leq i \leq n_T$$

$$n_T \geq n_T - i, \quad 1 \leq i \leq n_T$$

which is always satisfied, given $c_5 > 0$.

3.4.6. Conclusions of the Requirements for a Monotone Nondecreasing Policy.

The results obtained from the analysis performed in Sections 3.4.1 to 3.4.5 are summarized in the next theorem and corollaries:

Theorem 1

For the Markov Decision Model shown in Section 3.1, if inequalities (11), (12), (13), (14) and (15) are satisfied for the cost parameters, as well as inequality (21) for the return rate r , then a monotone nondecreasing policy is optimal.

Corollary 1

If inequalities (11) to (15) are satisfied, and also:

$$c_7 \leq c_1 + c_3 + c_4 \quad (22)$$

Then there is an optimal monotone nondecreasing policy for any $r \leq 1$.

Corollary 2

If inequalities (11) to (15) are satisfied and also:

$$c_5 - c_7 > c_7 - c_1 - c_3 - c_4 \quad (23)$$

Then there is an optimal monotone nondecreasing policy for any $r \leq 0.5$.

Inequality (22) implies that the unit cost of outsourcing RL functions is less or equal than the corresponding unit capacity cost of creating and keeping enough capacity to remanufacture one unit, including its reprocessing cost; i.e., the unit cost of developing

capacity and remanufacturing returns internally is greater than the unit outsourcing cost. If such situation takes place, then there exists an optimal monotone nondecreasing policy, regardless the value that the return rate takes.

On the other hand, inequality (23) implies that the opportunity (regret) cost $(c_5 - c_7)$ of not taking the outsourcing option and incurring the corresponding shortage for a particular unit, is greater than the opportunity cost $(c_7 - c_1 - c_3 - c_4)$ of taking the outsourcing option, instead of creating and keeping internal capacity to remanufacture such unit; i.e., considering that (as mentioned in section 3.1.5) the 3PRLP has infinite capacity, the regret of incurring a shortage when the outsourcing option was not taken, is greater than the regret of incurring the outsourcing cost instead of creating and using internal capacity. If such situation takes place, then there exists an optimal monotone nondecreasing policy for any RL system where the return rate is below 0.5.

The previous theorem and corollaries, as well as the fact that (as mentioned at the beginning of Chapter 3) the return rate in most industries is between zero and 0.3, approaching 0.5 only in some specific sectors (Rogers and Tibben-Lembke, 1999) imply that, the cases where $r \leq 0.5$ and inequality (23) is not satisfied, are of special interest, given that the threshold defined by (21) as well as the return rate r are below 0.5. In other words, there is no certainty about the existence of an optimal monotone nondecreasing policy in such cases.

3.5. Implications for the Suitability of Outsourcing for Scenarios with High Return Volume Variability.

In Chapter 2, the most important scenarios for RL systems were categorized according to the length of the product's life cycle, as well as the variability in the return volume. This qualitative study showed that the outsourcing option seems to be more suitable for managing returns of products with higher variability on the return volume and shorter life cycles. This was the basis to establish hypothesis 3, as well as research objective 4 for this dissertation.

In order to analyze this hypothesis and objective, consider the partial state ordering defined for the MDM proposed:

$$(k^1, w^1) \prec (k^2, w^2) \Leftrightarrow k^1 = k^2 \text{ and } w^1 < w^2$$

Then, the result of Theorem 1 implies that:

$$k^1 = k^2 \text{ and } w^1 < w^2 \Rightarrow a^*(k^1, w^1) \leq a^*(k^2, w^2)$$

where $a^*(k, w)$ was already defined in Section 3.2. Then, this corresponds to a MNDP. Throughout this section, we suppress the subscript t for simplicity.

The outsourcing threshold for each capacity level is defined as:

$$\theta(k) = \begin{cases} \min\{w : a^*(k, w) = 1\} & \text{if } \theta(k) \text{ exists} \\ \infty & \text{otherwise} \end{cases}$$

Lemma 2:

Suppose the conditions of Theorem 1 are satisfied. Let $\theta(k; r)$ be the value of $\theta(k)$ when the return rate is r . If $0 \leq r \leq r + \Delta_r \leq 1$ and $(c_3 - c_2 + c_4)\Delta_r \geq (c_5 - c_4)r$, then:

$$\theta(k; r) \geq \theta(k; r + \Delta_r) \quad (24)$$

Proof:

Let $w \equiv \theta(k; r)$, and let $R((k, w), a; r)$ be the value of $R((k, w), a)$ when the return rate is r . In order to prove (30), the next inequalities must be satisfied, given the relationships for r and Δ_r defined in terms of the cost parameters:

$$R((k, w), 0; r) \leq R((k, w), 1; r) \quad (25)$$

$$R((k, w), 0; r + \Delta_r) \leq R((k, w), 1; r + \Delta_r) \quad (26)$$

where (25) comes from the definition of w . Inequality (26) implies that the threshold is not greater than w when the return rate increases Δ_r ; i.e., the threshold does not increase when the return rate increases, as stated in (24).

Given that by definition of w , inequality (25) is satisfied, and that the right-hand sides in both inequalities are equal (they do not depend on r), it can be said by transitive property that inequality (26) will be satisfied as long as:

$$R((k, w), 0; r + \Delta_r) \leq R((k, w), 0; r), \quad (27)$$

where (27) can be rewritten as:

$$\begin{aligned} & c_1 \left((n(r + \Delta_r) - k)^+ - (nr - k)^+ \right) + c_2 \left((k - n(r + \Delta_r))^+ - (k - nr)^+ \right) + c_3 n \Delta_r \\ & + c_4 \left(E(\min(x_1, n(r + \Delta_r))) - E(\min(x_2, nr)) \right) \\ & + c_5 \left(E(\max(x_1 - n(r + \Delta_r), 0)) - E(\max(x_2 - nr, 0)) \right) > 0 \text{ where } x_1 \text{ Bin}(n, r + \Delta_r), \\ & x_2 \text{ Bin}(n, r) \end{aligned} \quad (28)$$

In particular, if the next elements are considered from (28):

$$\begin{aligned} & c_4 \left(E(\min(x_1, n(r + \Delta_r))) - E(\min(x_2, nr)) \right) + \\ & c_5 \left(E(\max(x_1 - n(r + \Delta_r), 0)) - E(\max(x_2 - nr, 0)) \right) \end{aligned} \quad (29)$$

From (13) and (14), it can be said for (29) that:

$$\begin{aligned} & c_4 \left(E(\min(x_1, n(r + \Delta_r))) - E(\min(x_2, nr)) \right) \\ & + c_5 \left(E(\max(x_1 - n(r + \Delta_r), 0)) - E(\max(x_2 - nr, 0)) \right) \\ & > c_4 \left[E(\min(x_1, n(r + \Delta_r))) + E(\max(x_1 - n(r + \Delta_r), 0)) \right] \\ & - c_5 \left[E(\min(x_2, nr)) + E(\max(x_2 - nr, 0)) \right] \\ & = c_4 E(x_1) - c_5 E(x_2) = c_4 n(r + \Delta_r) - c_5 nr \end{aligned} \quad (30)$$

Considering (30), inequality (28) can be analyzed under the next three cases:

- 1) $nr \geq k$
- 2) $nr < k, n(r + \Delta_r) \geq k$
- 3) $n(r + \Delta_r) < k$

In the first case, (28) is reduced to:

$$n\Delta_r(c_1 + c_3) + c_4n(r + \Delta_r) - c_5nr > 0,$$

which can be rewritten as:

$$(c_1 + c_3 + c_4)\Delta_r \geq (c_5 - c_4)r. \quad (31)$$

In the second case, (28) reduced to:

$$c_1(nr + n\Delta_r - k) + n\Delta_r(c_3 - c_2) + c_4n(r + \Delta_r) - c_5nr > 0,$$

which can be rewritten as:

$$c_1(nr + n\Delta_r - k) + (c_3 - c_2 + c_4)\Delta_r \geq (c_5 - c_4)r. \quad (32)$$

Finally, in the third case (28) is reduced to:

$$n\Delta_r(c_3 - c_2) + c_4n(r + \Delta_r) - c_5nr > 0,$$

which can also be rewritten as:

$$(c_3 - c_2 + c_4)\Delta_r \geq (c_5 - c_4)r. \quad (33)$$

However, because of (11) and given that $nr < k \leq n(r + \Delta_r)$ in the second case, inequalities (31) and (32) will be satisfied as long as (33) is satisfied. Then, this inequality represents a sufficient condition to satisfy (24). This completes the proof.

Lemma 3:

Suppose the conditions of Theorem 1 are satisfied. Let $q((nr, w_l)(k, w), 0; r)$ be the value of $q((nr, w_l)(k, w), 0)$ when the return rate is r . If $0 \leq r + \Delta_r \leq 0.5$ then:

$$q((n(r + \Delta_r), w_l)(k, w), 0; r + \Delta_r) \geq q((nr, w_l)(k, w), 0; r) \quad (34)$$

Proof:

Inequality (34) can be rewritten as:

$$\sum_{j=w_l}^n \binom{n}{j} (r + \Delta_r)^j (1 - r - \Delta_r)^{n-j} \geq \sum_{j=w_l}^n \binom{n}{j} r^j (1 - r)^{n-j}$$

which is equivalent to:

$$P\{x_1 > w_l - 1\} \geq P\{x_2 > w_l - 1\} \text{ for } w_l = \{1, 2, \dots, n\} \quad (35)$$

where x_1 is binomial with parameters n and $r + \Delta_r$ and x_2 is binomial with parameters n and r . Considering such distributions, Rao (1952) states that $P\{x_1 \leq m\} \leq P\{x_2 \leq m\}$ for any $m \in \{0, 1, \dots, n\}$. This implies that:

$$P\{x_1 > m\} \geq P\{x_2 > m\} \text{ for } m \in \{0, 1, \dots, n\} \quad (36)$$

Given that (35) is equivalent to (36), it can be concluded that (34) is satisfied. This completes the proof.

Based on Lemmas 2 and 3, the next theorem can be stated:

Theorem 2:

Suppose the conditions of Theorem 1, Lemma 2 and Lemma 3 are satisfied. Then:

$$q((n(r + \Delta_r), \theta(k; r + \Delta_r)) | (k, w), 0; r + \Delta_r) \geq q((nr, \theta(k; r)) | (k, w), 0; r)$$

Proof:

From Lemma 2 it can be said that:

$$\theta(k; r) \geq \theta(k; r + \Delta_r)$$

which implies that:

$$q((nr, \theta(k; r + \Delta_r)) | (k, w), 0; r) \geq q((nr, \theta(k; r)) | (k, w), r) \quad (37)$$

From Lemma 3 it can also be said that:

$$q((n(r + \Delta_r), \theta(k; r + \Delta_r)) | (k, w), 0; r + \Delta_r) \geq q((nr, \theta(k; r + \Delta_r)) | (k, w), r) \quad (38)$$

Then, we have by the transitive property that:

$$q((n(r + \Delta_r), \theta(k; r + \Delta_r)) | (k, w), 0; r + \Delta_r) \geq q((nr, \theta(k; r)) | (k, w), r)$$

which completes the proof.

Theorem 2 implies that, as stated in hypothesis 3, the suitability of the outsourcing option increases when the return rate increases. This comes not only from the fact that the probability of crossing the corresponding threshold that determines the optimality of the outsourcing option increases, but also from the fact that the value for such threshold does not increase.

Then, given that the variability on the return volume increases as the return rate increases (for any value below 0.5), it can be concluded that, as mentioned in hypothesis

3, outsourcing becomes a more suitable option for products with greater variability in its return volume.

This situation is supported by the fact that (as shown in Lemma 2) in most cases, the expected reward for $a = 0$ decreases as the return rate, and in consequence the variability in the return volume, increases. Such decrease for the reward when performing RL activities internally causes the threshold that determines the optimality of the outsourcing option not to increase. Even more, as the return rate increases, such threshold may decrease, which increments the size of the set of states where outsourcing is optimal. This situation will be supported by two numerical examples that will be shown in the next section.

3.5.1. Numerical Example.

In order to show the influence of higher variability of the return volume on the suitability of an outsourcing option, consider a particular scenario defined by the next parameters:

$$\begin{array}{ll}
 L = 4 & c_3 = 3 \\
 W = 1 & c_4 = 8 \\
 & c_5 = 24 \\
 c_1 = 1 & c_6 = 2 \\
 c_2 = 1 & c_7 = 13
 \end{array} \tag{39}$$

As well as the next sales function:

$$s_t = \begin{cases} \frac{2M}{L}t, & t = 1, 2, \dots, L/2 \\ M - \frac{2M}{L}(t - L/2 - 1), & t = L/2 + 1, \dots, L \end{cases} \tag{40}$$

where $M = 3$. The values for the cost parameters satisfy conditions (11) to (15) as well as (22); i.e., there is an optimal monotone nondecreasing policy for any $r \leq 1$.

Table 8 shows the values for the threshold in each set of states, for $r = \{0.2, 0.3, 0.4, 0.5\}$, as well as the probability $q_i((k_t, w_t)|(k_{t-1}, w_{t-1}), a)$ that such threshold (defined as w_t) is crossed in each case, where (as mentioned in section 3.3.2):

$$q_i((k_t, w_t)|(k_{t-1}, w_{t-1}), a) = \sum_{w_t=w_i}^{n_{t-1}} p_i((k_t, w_t)|(k_{t-1}, w_{t-1}), a)$$

These values were obtained by creating a Matlab program (see Appendix 1) whose inputs are $r, L, W, c_1, c_2, c_3, c_4, c_5, c_6, c_7$, as well as the sales volume s_t during the horizon analysis.

Based on this information, the program computes the possible states and orders them according to the criteria defined. The program also computes the amount of units n_t outstanding in the market for each state, as well as the corresponding transition probabilities and expected costs for $a=0$ and $a=1$. The terminal costs are also obtained.

Based on this, the program solves the MDM by using backward induction, and shows the optimal action to take at each decision epoch. At the end, the results are sent to an Excel file (see Appendix 2 for a resume of the outputs), where all the information is showed.

As it can be identified in this table, a greater variability on the return volume (greater r) increases the probability of crossing the corresponding threshold in each set of states; i.e., there is a greater probability that outsourcing ($a=1$) will be the optimal action to take.

This implies that, as mentioned in section 2.2, greater variability in the return volume increases the uncertainty about the volume of units put into the corresponding RL system, which forces the firm to follow an outsourcing strategy, and take advantage of the economies of scale by involving a 3PRLP in managing returned items.

Table 8. Value of the threshold and the probability of crossing it for $r = \{0.2, 0.3, 0.4, 0.5\}$ ⁽¹⁾

t	States k_t	$r = 0.2$		$r = 0.3$		$r = 0.4$		$r = 0.5$	
		w_t	q_t	w_t	q_t	w_t	q_t	w_t	q_t
1	$0r$	-	0	-	0	-	0	0	1
2	$2r$	-	0	-	0	-	0	0	1
3	$5r$	-	0	-	0	5	0.0124	1	0.922
3	$4r$	-	0	-	0	-	0	3	0.524
3	$3r$	-	0	-	0	-	0	3	0.784
4	$8r$	-	0	8	0.0006	6	0.049	4	0.405
4	$7r$	-	0	-	0	7	0.188	4	0.580
4	$6r$	-	0	-	0	7	0.040	5	0.455
4	$5r$	-	0	-	0	8	0.010	5	0.663
4	$4r$	-	0	-	0	8	0.025	6	0.524
4	$3r$	-	0	-	0	8	0.064	7	0.352

(1): w_t = Threshold above which outsourcing is optimal (“-” means there is no threshold; i.e., $a=0$ is optimal for all states in that group).

q_t = Probability that the threshold is crossed.

Now, in order to show the influence of higher variability in the return volume and a shorter product’s life cycle, consider the next two cases:

$$\begin{aligned}
 \text{Case 1:} & \quad r = 0.3, L = 5 \\
 \text{Case 2:} & \quad r = 0.5, L = 4
 \end{aligned} \tag{41}$$

where both of them are defined by the cost parameters and value for W shown in (39), as well as sales function (40) with $M = 3$. Tables 9 and 10 show the results for the two cases considered.

By comparing both cases, it can be identified that when the variability on the return volume increases and the length of the life cycle decreases, the probability of being above the threshold that determines the optimality of the outsourcing option increases; i.e., outsourcing is a more suitable option in such situation.

Table 9. Value of the threshold and the probability of crossing it for Case 1: $r = 0.3$, $L = 5$ ⁽¹⁾

t	k_t	w_t	q_t
1	0	-	0
2	0.3	-	0
3	0.9	-	0
3	0.6	-	0
4	2.1	7	0.0002
4	1.8	7	0.0007
4	1.5	-	0
4	1.2	-	0
5	2.7	8	0.0004
5	2.4	8	0.0012
5	2.1	9	0.0002
5	1.8	9	0.0007
5	1.5	9	0.002
5	1.2	9	0.0081
5	0.9	9	0.027
5	0.6	-	0

(1): w_t = Threshold above which outsourcing is optimal (“-” means there is no threshold; i.e., $a=0$ is optimal for all states in that group).

q_t = Probability that the threshold is crossed.

Then, given $r \leq 0.5$, the threshold that determines the suitability of the outsourcing option for the Markov Decision Model developed is easily crossed in the scenario where the variability on the return volume is greater (greater r), and the length L of the product life cycle is shorter.

This supports the conclusions regarded from the qualitative analysis developed on chapter 2, as well as hypothesis 3 and a numerical example for research objective 4. Due to a high variability in the rate of returns, it may not be economically feasible at all for a firm to develop its own RL facilities, given that the amount of units to be returned will be significantly uncertain over time, and the required capacity will be changing constantly.

Table 10. Value of the threshold and the probability of crossing it for Case 2: $r = 0.5$, $L = 4$ ⁽¹⁾

t	k_t	w_t	q_t
1	0	0	1
2	1	0	1
3	2.5	1	0.968
3	2	3	0.687
3	1.5	3	0.875
4	4	4	0.636
4	3.5	4	0.773
4	3	5	0.656
4	2.5	5	0.812
4	2	6	0.687
4	1.5	7	0.5

(1): w_t Threshold above which outsourcing is optimal (“-” means there is no threshold; i.e., $a=0$ is optimal for all states in that group).

q_t = Probability that the threshold is crossed.

The complexity of this situation increases when the life cycle for this type of products is extremely short, which requires quick but adequate decisions for these RL systems, in order to efficiently respond to such changing conditions. This can effectively be accomplished by involving a 3PRLP, which specializes in these activities, and can take advantage of the economies of scale to convert RL functions in a profit-creating activity into the closed-loop chain.

4. Conclusions and Future Work.

4.1. Verifying Research Hypotheses & Objectives.

RL is a considerable improvement area for any firm when focused correctly. By considering two critical factors, the length of the product's life cycle and the variability in the return volume, a characterization of RL networks was initially proposed in this document, which corresponds to the answer of hypothesis 1 in this research. The convenience of using this categorization to analyze every RL channel was also shown, as well as the categories where some of the most important RL networks in the U.S. market can be classified. Some 3PRLPs that actually offer their services in some of the proposed scenarios were described too. Even more, it was identified that most of them offer their services for companies which products have a relatively short life cycle and high variability in its return volume. This corresponds to the answer of the second research hypothesis.

The third research hypothesis was addressed by developing a Markov Decision Model (MDM) for RL systems, which models the RL outsourcing decision, under the implied uncertainty for the return volume. It considers several elements which are critical in defining the characteristics of a RL network, such as the rate of return experienced, the length of the product life cycle, the sales behavior, the particular RL costs incurred, as well as the length of time defined for the existence of that RL system. In particular, the length of the product life cycle, the cost parameters, the sales function defined and the rate of return considered, are modeling the scenario of interest; i.e., the length of the horizon analysis in the MDM is determined by such life cycle; while the uncertainty implied in the MDM is represented by the expected amount of returned units, which is defined by the outstanding units in the market and the rate of return considered.

The conditions for the existence of an optimal monotone nondecreasing policy were also shown, where it was verified that such a policy will exist as long as a set of assumptions on the cost parameters is satisfied, and the return rate is below a bound defined in terms

of such cost parameters. Even more, there are some special instances where an optimal monotone nondecreasing policy exists, regardless the value for the return rate.

The existence of an optimal monotone nondecreasing policy implies the presence of a threshold above which, it is optimal to follow an outsourcing strategy for the RL system; otherwise, to continue performing the RL activities internally. Such threshold was defined in terms of a partial ordering for the system states, where given a fixed capacity at a decision epoch, the states are ordered according to the cumulative returned units, such that if that volume goes above a particular level, then it is convenient to follow an outsourcing strategy and take advantage of the economies of scale implied by involving a 3PRLP in managing the returns, which has RL as its core function.

The hypothesis that outsourcing is a more suitable option for scenarios with greater variability on the return volume was also supported, by showing analytically the increment in the probability of crossing the threshold that determines outsourcing optimality when the variability in the return volume increases. It was also showed how such threshold does not increase when the return volume variability increases. Even more, it may decrease as such variability increases, which also increases the probability of crossing it.

As a support to this analysis, two sets of scenarios were showed using a Matlab program developed for this instance. In the first set, the rate of returns was increased while keeping everything else fixed. The results showed that outsourcing is more suitable when the rate of returns (and in consequence the variability in the return volume) is greater.

Finally, the second set contained two different scenarios with the same cost parameters and sales function, but with different variability in the return volume and length of the product's life cycle. In the second scenario, a greater variability and shorter life cycle existed, and (as expected) outsourcing was a more suitable option in this scenario than in the first one.

4.2. Future Work.

Considering the previous conclusions, it can be said that all of the hypotheses and objectives stated on this dissertation were satisfied, except research objective 4, which was partially achieved. This can represent the basis for future research to be developed based on this dissertation.

Even though the existence of an optimal monotone nondecreasing policy was shown, the influence on the suitability of the outsourcing option was analytically proved just for the return volume variability, but not for the length of the product life cycle, whose influence was just showed numerically. A set of examples which support objective 4 were developed, but such analytical proof for the influence of the life cycle length on outsourcing suitability represents a future research area to consider. The main challenge for such analysis is the difference on the size of the sets of ordered states obtained for each case.

This comes from the fact that the size of the state space at each decision epoch is determined by the sales function of the product analyzed, as well as the length of the lifecycle. The greater the sales volume is in each period, the greater the state space in the next period will be. Considering this, the relationship between the sales volume, the size of the state space and the value for the threshold, is a relevant area of research to consider too. Also, the probability of being above such threshold is determined by the size of the state space, and such probability represents a basis for evaluating the suitability of the outsourcing option for a particular RL system.

Another area of research can be defined on identifying the requirements for the existence of an optimal monotone nondecreasing policy, when the returns follow a probability distribution different than the one described in this dissertation. The influence of a different stochastic behavior for the returns (according to a particular scenario of interest) can be considered, which will represent the basis for evaluating the five conditions required for such kind of policy.

Finally, future research can also be focused on answering the next questions: what if the rate of returns and/or the RL costs are not constant during the product's life cycle?, what happens when the RL capacity is not adjusted to the expected returns, and/or the firm does not have the capability of adjusting such capacity at the end of any period?, what happens when the 3PRLP fails to perform adequately, such that the outsourcing option does not remain for the rest of the planning horizon?, what if profits for RL activities are also considered?. All of these questions represent an interesting basis for future research focused also on evaluating an outsourcing option for a RL system.

Appendix 1: Matlab Program

```

clc
L=4;
W=1;
T=L+W;
r=0.4;
C=[1 1 3 8 24 2 13];
S=[2 3 3 2 0 0 0 0 0];
RL=zeros(100,11);
APP=1;
APR=1;
APH=0;
while RL(APP,1)< T
    APH=0;
    APH=RL(APP,2)+1;
    for i=1:APH
        APR=APR+1;
        RL(APR,1) = RL(APP,1)+1;
        RL(APR,5) = S( RL(APR,1) );
        RL(APR,3) = RL(APP,2)*r;
        RL(APR,4) = RL(APP,4)+i-1;
        RL(APR,2) = RL(APP,2) + RL(APR,5) - ( RL(APR,4) - RL(APP,4) );
        RL(APR,6) = prod(1:RL(APP,2)) / (prod(1:(RL(APP,2)-(RL(APR,4)-
RL(APP,4)))) * prod(1:(RL(APR,4)-RL(APP,4)))) * (r^(RL(APR,4)-
RL(APP,4))) * (1-r)^(RL(APP,2)-(RL(APR,4)-RL(APP,4)))));
        RL(APR,7) = APP;
        if (RL(APR,4)-RL(APP,4))<=RL(APR,3)
            RL(APP,9)=RL(APP,9)+C(4) * (RL(APR,4)-RL(APP,4)) * RL(APR,6);
        else
            RL(APP,9)=RL(APP,9)+C(4) * RL(APR,3) * RL(APR,6) + C(5) * ((RL(APR,4) -
            RL(APP,4)) - RL(APR,3)) * RL(APR,6);
        end
    end
    APP=APP+1;
end
for i=1:APP-1
    if RL(i,2)*r>RL(i,3);
        RL(i,13)=C(1) * (RL(i,2) * r - RL(i,3));
        RL(i,9)=RL(i,9)+RL(i,13);
    else
        RL(i,13)=C(2) * (RL(i,3) - RL(i,2) * r);
        RL(i,9)=RL(i,9)+RL(i,13);
    end
    RL(i,13)=RL(i,13)+C(3) * RL(i,2) * r;
    RL(i,9)=RL(i,9)+C(3) * RL(i,2) * r;
end
for i=2:APP-1
    FS=0;
    for j=RL(i,1):L;
        FS = FS + S(j+1);
    end
    RL(i,10) = C(7) * ((RL(i,2)+FS)) + C(6) * RL(i,3);
end
for j=1:APR
    if RL(j,1)>T-1;
        RL(j,8)=C(6) * RL(j,3) + C(5) * RL(j,2) * r;
    else RL(j,8)=0;
    end
end

```

```

end
APP=APP-1;
APH=APP+1;
while RL(APP,1)>T-2
  while RL(APH,7)-APP<0
    APH=APH+1;
  end
  for j=APH:APH+RL(APP,2)
    if (RL(j,4)-RL(APP,4))>RL(j,3)
      BI=(C(4)*RL(j,3)+C(5)*(RL(j,4)-RL(APP,4)-
RL(j,3))+RL(j,8))*RL(j,6);
    else
      BI=(C(4)*(RL(j,4)-RL(APP,4))+RL(j,8))*RL(j,6);
    end
    RL(APP,11)=RL(APP,11)+BI;
  end
  RL(APP,11)=RL(APP,11)+RL(APP,13);
  if RL(APP,11)-RL(APP,10)>0
    RL(APP,12)=RL(APP,11);
    RL(APP,14)=0;
  else
    RL(APP,12)=RL(APP,10);
    RL(APP,14)=1;
  end
  APP=APP-1;
  APH=APP+1;
end
while RL(APP,1)>0
  while RL(APH,7)-APP<0
    APH=APH+1;
  end
  for j=APH:APH+RL(APP,2)
    if (RL(j,4)-RL(APP,4))>RL(j,3)
      BI=(C(4)*RL(j,3)+C(5)*(RL(j,4)-RL(APP,4)-
RL(j,3))+RL(j,12))*RL(j,6);
    else
      BI=(C(4)*(RL(j,4)-RL(APP,4))+RL(j,12))*RL(j,6);
    end
    RL(APP,11)=RL(APP,11)+BI;
  end
  RL(APP,11)=RL(APP,11)+RL(APP,13);
  if RL(APP,11)-RL(APP,10)>0
    RL(APP,12)=RL(APP,11);
    RL(APP,14)=0;
  else
    RL(APP,12)=RL(APP,10);
    RL(APP,14)=1;
  end
  APP=APP-1;
  APH=APP+1;
end
RL
wklwrite ('MDMOUTPUT',RL,2,0)

```

Appendix 2: Results from the outputs of the Matlab Program (r=0.2, 0.3, 0.4 and 0.5)

w_l	Value for w_t (given k_t) above the one outsourcing (a=1) is optimal.
q_t	Probability that the threshold is crossed.

t	n_t	k_t	w_t	s_t	p_t	r=0.2		r=0.3		r=0.4		r=0.5	
						w_l	q_t	w_l	q_t	w_l	q_t	w_l	q_t
1	2	0	0	2	1	0	0	0	0	0	0	1	1
2	5	0.8	0	3	0.36	0	0	0	0	0	0	1	1
2	4	0.8	1	3	0.48	0		0		0		1	
2	3	0.8	2	3	0.16	0		0		0		1	
3	8	2	0	3	0.078	0	0	0	0	0	0.0124	0	0.92
3	7	2	1	3	0.259	0		0		0		1	
3	6	2	2	3	0.346	0		0		0		1	
3	5	2	3	3	0.23	0		0		0		1	
3	4	2	4	3	0.077	0		0		0		1	
3	3	2	5	3	0.01	0		0		1		1	
3	7	1.6	1	3	0.13	0	0	0	0	0	0	0	0.52
3	6	1.6	2	3	0.346	0		0		0		0	
3	5	1.6	3	3	0.346	0		0		0		1	
3	4	1.6	4	3	0.154	0		0		0		1	
3	3	1.6	5	3	0.026	0		0		0		1	
3	6	1.2	2	3	0.216	0	0	0	0	0	0	0	0.78
3	5	1.2	3	3	0.432	0		0		0		1	
3	4	1.2	4	3	0.288	0		0		0		1	
3	3	1.2	5	3	0.064	0		0		0		1	
4	10	3.2	0	2	0.017	0	0	0	0.006	0	0.049	0	0.40
4	9	3.2	1	2	0.09	0		0		0		0	
4	8	3.2	2	2	0.209	0		0		0		0	
4	7	3.2	3	2	0.279	0		0		0		0	
4	6	3.2	4	2	0.232	0		0		0		1	
4	5	3.2	5	2	0.124	0		0		0		1	
4	4	3.2	6	2	0.041	0		0		1		1	
4	3	3.2	7	2	0.008	0		0		1		1	
4	2	3.2	8	2	7E-04	0		1		1		1	
4	9	2.8	1	2	0.028	0	0	0	0	0	0.188	0	0.58
4	8	2.8	2	2	0.131	0		0		0		0	
4	7	2.8	3	2	0.261	0		0		0		0	
4	6	2.8	4	2	0.29	0		0		0		1	
4	5	2.8	5	2	0.194	0		0		0		1	
4	4	2.8	6	2	0.077	0		0		0		1	
4	3	2.8	7	2	0.017	0		0		1		1	
4	2	2.8	8	2	0.002	0		0		1		1	
4	8	2.4	2	2	0.047	0	0	0	0	0	0.040	0	0.45
4	7	2.4	3	2	0.187	0		0		0		0	
4	6	2.4	4	2	0.311	0		0		0		0	
4	5	2.4	5	2	0.276	0		0		0		1	
4	4	2.4	6	2	0.138	0		0		0		1	
4	3	2.4	7	2	0.037	0		0		1		1	
4	2	2.4	8	2	0.004	0		0		1		1	
4	7	2	3	2	0.078	0	0	0	0	0	0.010	0	0.66

4	6	2	4	2	0.259	0		0		0		0	
4	5	2	5	2	0.346	0		0		0		1	
4	4	2	6	2	0.23	0		0		0		1	
4	3	2	7	2	0.077	0		0		0		1	
4	2	2	8	2	0.01	0		0		1		1	
4	6	1.6	4	2	0.13	0	0	0	0	0	0.025	0	0.52
4	5	1.6	5	2	0.346	0		0		0		0	
4	4	1.6	6	2	0.346	0		0		0		1	
4	3	1.6	7	2	0.154	0		0		0		1	
4	2	1.6	8	2	0.026	0		0		1		1	
4	5	1.2	5	2	0.216	0	0	0	0	0	0.064	0	0.35
4	4	1.2	6	2	0.432	0		0		0		0	
4	3	1.2	7	2	0.288	0		0		0		1	
4	2	1.2	8	2	0.064	0		0		1		1	

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